Noise-induced de-synchronization in dynamic stochastic consumption model

Jochen Jungeilges [1], Tatyana Perevalova [2]

[1] University of Agder, School of Business and Law, Department of Economics and Finance, Kristiansand S, Norway
[2] University of Pisa, Department of Economics and Management, Pisa, Italy

28.05.2025

PODE25, Cartagena, Spain

The Model

$$
\begin{aligned}
x_{1t+1} &= \frac{b_1}{p_x p_y}\left(\alpha_1 x_{1t}(b_1 - p_x x_{1t}) + D_{12} x_{2t}(b_2 - p_x x_{2t})\right) + \varepsilon \xi_{1t} \\
x_{2t+1} &= \frac{b_2}{p_x p_y}\left(\alpha_2 x_{2t}(b_2 - p_x x_{2t}) + D_{21} x_{1t}(b_1 - p_x x_{1t})\right) + \varepsilon \xi_{2t}
\end{aligned}
\tag{1}
$$

x_{it}	units of a commodity x consumed by individual i at time t
b_j	exogenous income of individual j
$p = (p_x, p_y)$	prices of consumption goods x and y
α_j	learning parameter
$D_{jj'}$	influence parameters
ε	noise intensity
ξ_{it}	Gaussian noise (iid)

$\varepsilon = 0$ deterministic model

$\varepsilon \neq 0$ stochastic model with additive noise

Constraints on the parameter space

Feasible phase region

If $\alpha_1 b_1^2 + D_{12} b_2^2 < 4 p_x p_y, \quad \alpha_2 b_2^2 + D_{21} b_1^2 < 4 p_x p_y$ holds, then $f(S) \subset S$ where

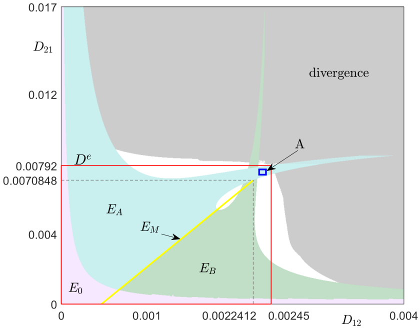
$$S = \left(0, \frac{b_1}{p_x}\right) \times \left(0, \frac{b_2}{p_x}\right) \tag{2}$$

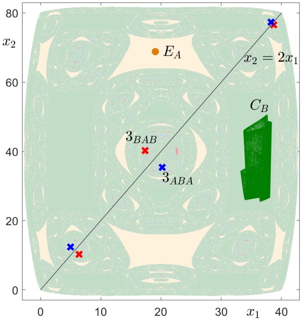
is the feasible phase region.

Economic environment: $p = (p_x, p_y) = \left(\frac{1}{4}, 1\right), \; b = (b_1, b_2) = (10, 20)$

1. $D_{12} < 0.25(0.01 - \alpha_1), D_{12} < 4(0.0025 - \alpha_2), \; S = (0, 40) \times (0, 80)$
2. Fix learning parameters $\alpha_1 = 0.0002$, $\alpha_2 = 0.00052$
3. $D^e = \{(D_{12}, D_{21}) \mid 0 \le D_{12} \le 0.00245 \wedge 0 \le D_{21} \le 0.00792\}$

2D bifurcation diagram





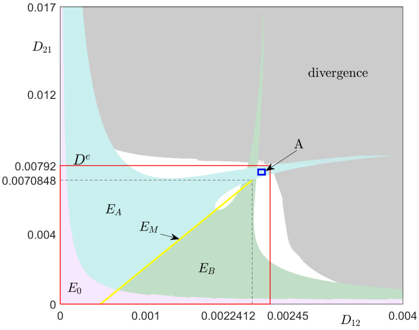
red line $\quad D^e = \{(D_{12}, D_{21}) \mid 0 \leq D_{12} \leq 0.00245 \wedge 0 \leq D_{21} \leq 0.00792\}$

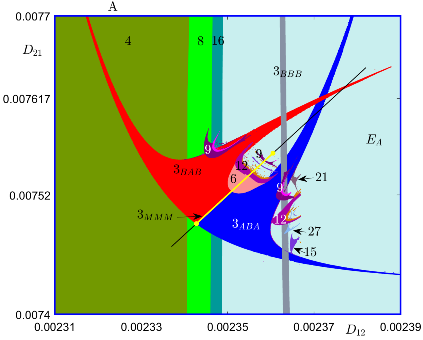
yellow line $\quad D_{21} = (D_{12} - \alpha_2)\frac{b_2^2}{b_1^2} + \alpha_1$[1], so that

$$f : x \mapsto f(x) = \frac{b_1^2\alpha_1 + b_2^2 D_{12}}{p_x p_y} x \left(1 - \frac{p_x}{b_1} x\right).$$

[1] Ekaterinchuk E., Jungeilges J., Ryazanova T., Sushko I. (2018). Dynamics of a minimal consumer network with bi-directional influence. Communications in Nonlinear Science and Numerical Simulation, 58, 107-118.

2D bifurcation diagram





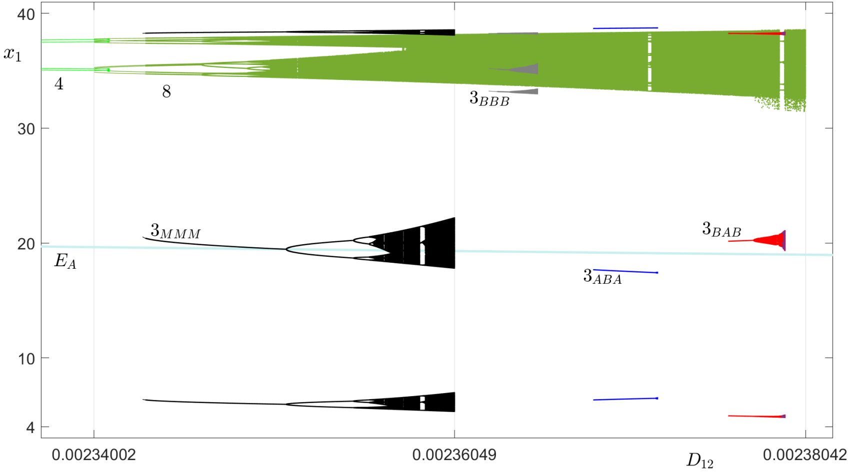
red line $\quad D^e = \{(D_{12}, D_{21}) \mid 0 \le D_{12} \le 0.00245 \wedge 0 \le D_{21} \le 0.00792\}$

yellow line $\quad D_{21} = (D_{12} - \alpha_2)\frac{b_2^2}{b_1^2} + \alpha_1$[1], so that

$$f : x \mapsto f(x) = \frac{b_1^2\alpha_1 + b_2^2 D_{12}}{p_x p_y} x \left(1 - \frac{p_x}{b_1} x\right).$$

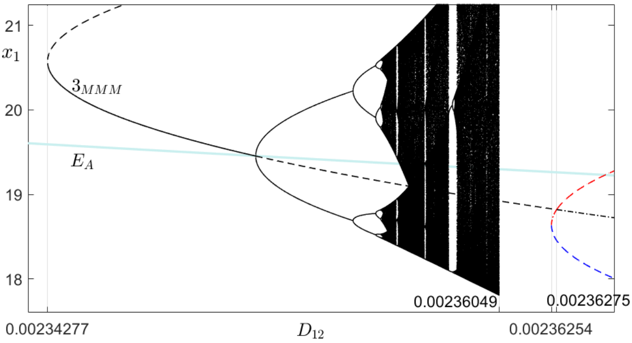
[1] Ekaterinchuk E., Jungeilges J., Ryazanova T., Sushko I. (2018). Dynamics of a minimal consumer network with bi-directional influence. Communications in Nonlinear Science and Numerical Simulation, 58, 107-118.

1D bifurcation diagram for $D_{21} = (D_{12} - \alpha_2)\frac{b_2^2}{b_1^2} + \alpha_1$

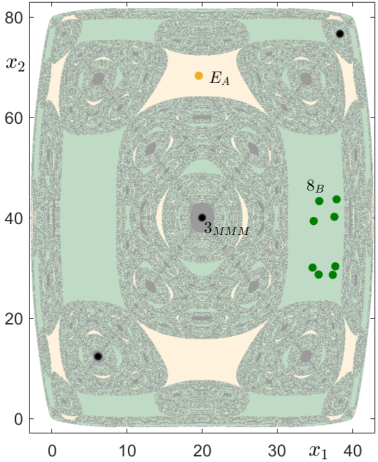


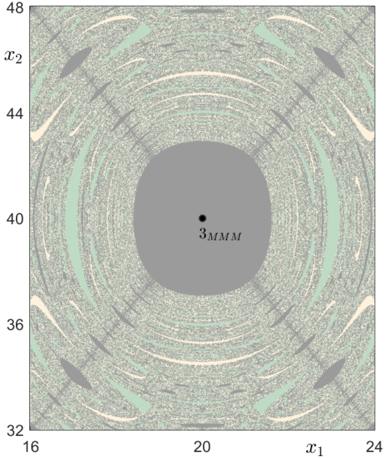
1D bifurcation diagram for $D_{21} = (D_{12} - \alpha_2)\frac{b_2^2}{b_1^2} + \alpha_1$

Synchronized attractors such that $x_2 = 2x_1$ (black)



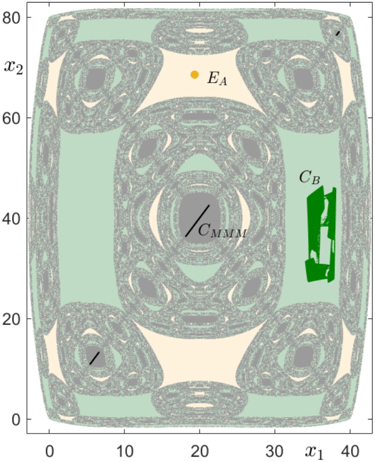
Attractors for $D_{12} = 0.00234492$, $D_{21} = 0.00749968$

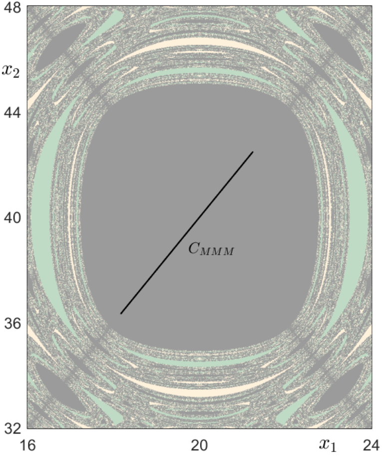




Here, for cycle 3_{MMM} holds $x_2 = 2x_1$

Attractors for $D_{12} = 0.002358,\ D_{21} = 0.007552$

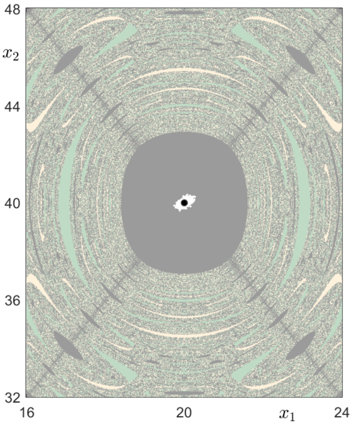


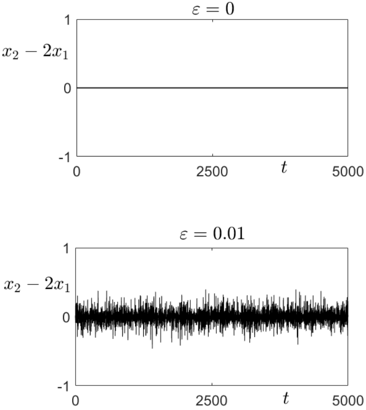


Here, for chaotic attractor C_{MMM} holds $x_2 = 2x_1$

Noise-induced de-synchronization

for $D_{12} = 0.00234492$, $D_{21} = 0.00749968$

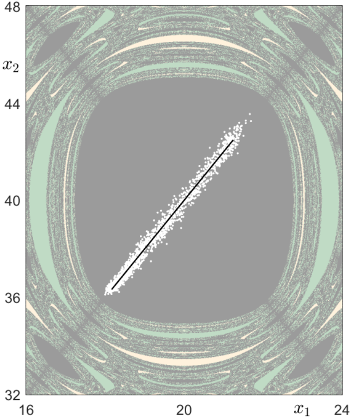


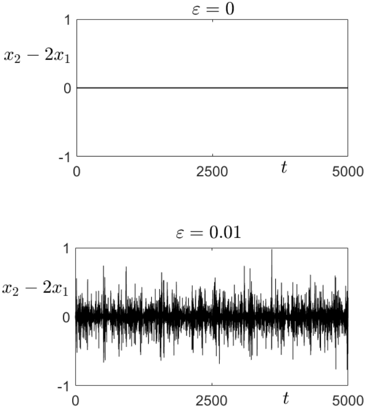


black $\varepsilon = 0$, white $\varepsilon = 0.01$

Noise-induced de-synchronization

for $D_{12} = 0.002358$, $D_{21} = 0.007552$





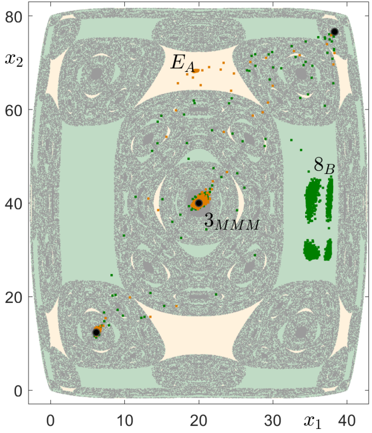
black $\varepsilon = 0$, white $\varepsilon = 0.01$

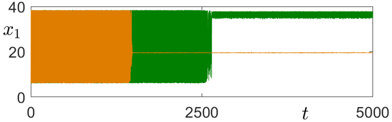
Noise-induced transition $3_{MMM} \to E_A$ or $\to 8_B$

for $D_{12} = 0.00234492$, $D_{21} = 0.00749968$ with $\varepsilon = 0.05$

Noise-induced transition $3_{MMM} \to E_A$ or $\to 8_B$

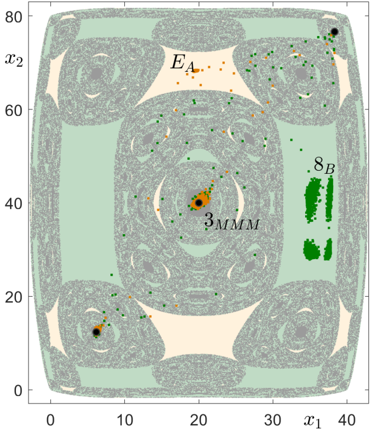
for $D_{12} = 0.00234492$, $D_{21} = 0.00749968$ with $\varepsilon = 0.05$

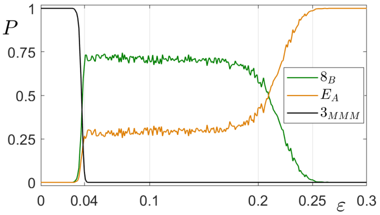


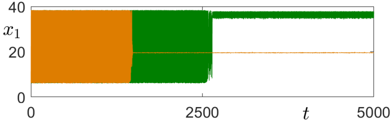


Noise-induced transition $3_{MMM} \to E_A$ or $\to 8_B$

for $D_{12} = 0.00234492$, $D_{21} = 0.00749968$ with $\varepsilon = 0.05$

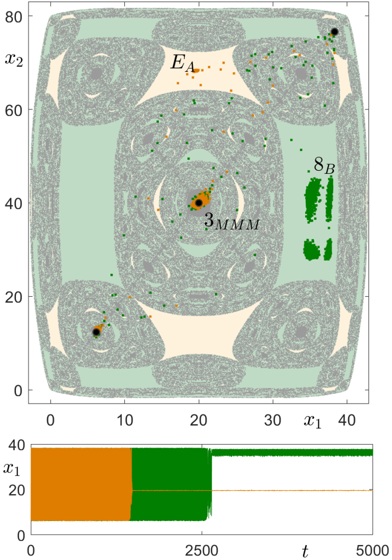


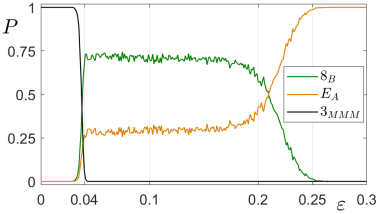


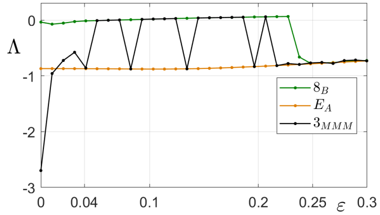


Noise-induced transition $3_{MMM} \to E_A$ or $\to 8_B$

for $D_{12} = 0.00234492$, $D_{21} = 0.00749968$ with $\varepsilon = 0.05$

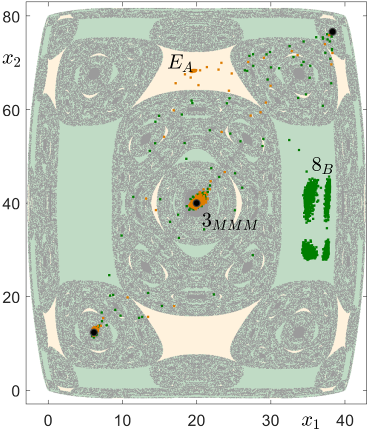


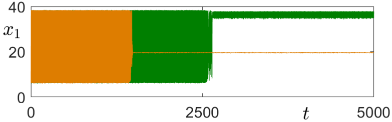


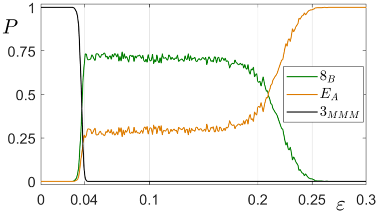


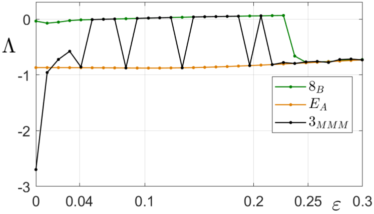
Noise-induced transition $3_{MMM} \to E_A$ or $\to 8_B$

for $D_{12} = 0.00234492$, $D_{21} = 0.00749968$ with $\varepsilon = 0.05$





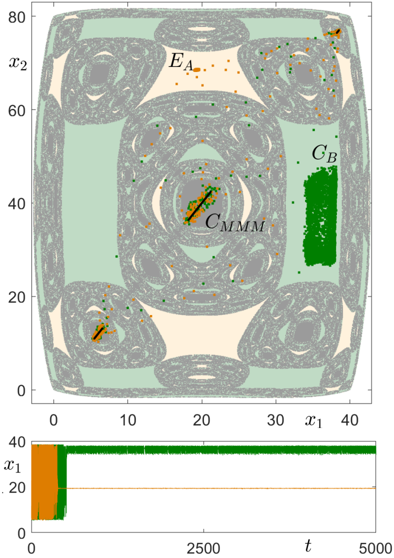




Stochastic D-bifurcations

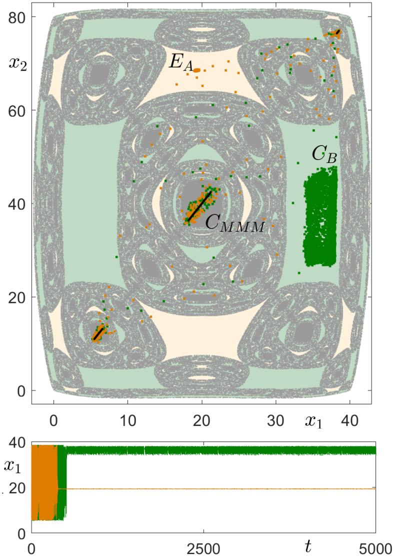
Noise-induced transition $C_{MMM} \to E_A$ or $\to C_B$

for $D_{12} = 0.002358$, $D_{21} = 0.007552$ with $\varepsilon = 0.05$



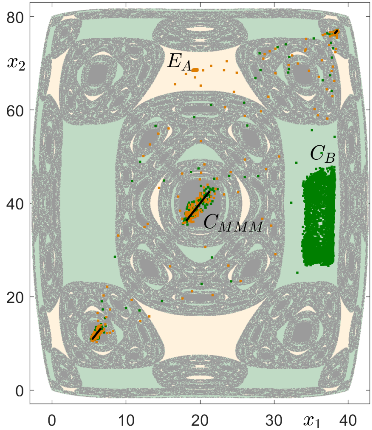
Noise-induced transition $C_{MMM} \to E_A$ or $\to C_B$

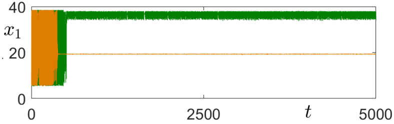
for $D_{12} = 0.002358$, $D_{21} = 0.007552$ with $\varepsilon = 0.05$

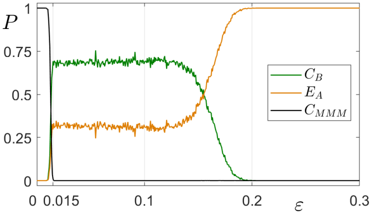


Noise-induced transition $C_{MMM} \to E_A$ or $\to C_B$

for $D_{12} = 0.002358$, $D_{21} = 0.007552$ with $\varepsilon = 0.05$

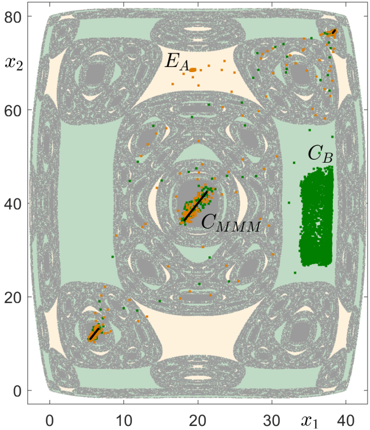


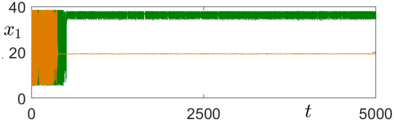


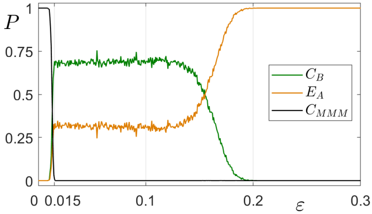


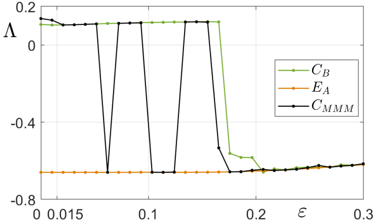
Noise-induced transition $C_{MMM} \to E_A$ or $\to C_B$

for $D_{12} = 0.002358$, $D_{21} = 0.007552$ with $\varepsilon = 0.05$

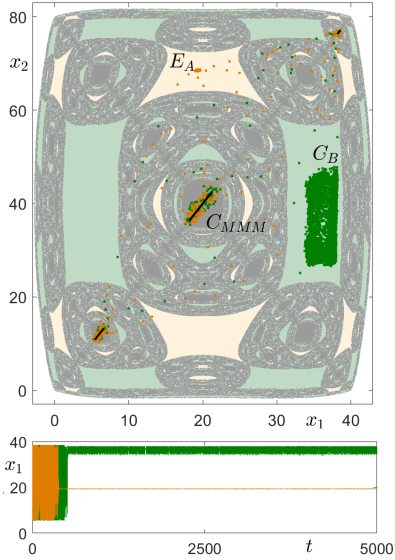


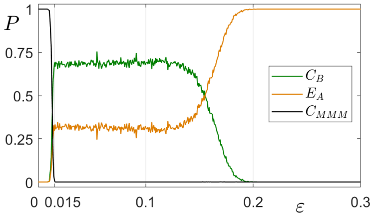


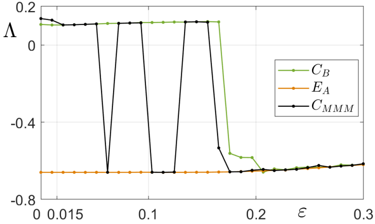




for $D_{12} = 0.002358$, $D_{21} = 0.007552$ with $\varepsilon = 0.05$



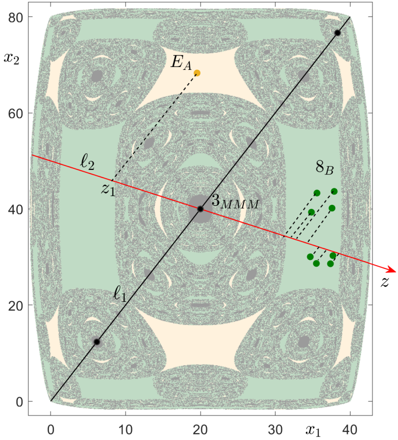




Stochastic D-bifurcations

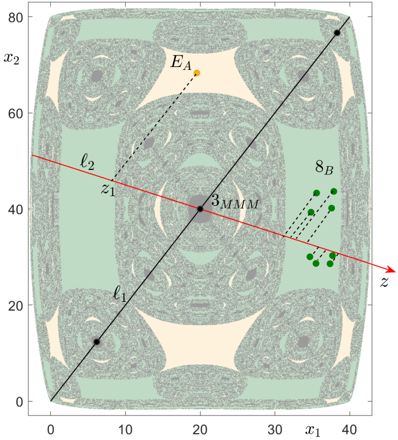
Noise-induced transition

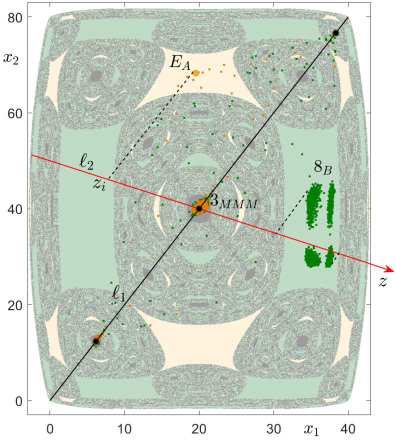
$$\ell_1 : x_2 = 2x_1, \qquad \ell_2 : x_2 = -\frac{1}{2}x_1 + 50 \qquad \ell_1 \perp \ell_2$$



Noise-induced transition

$\ell_1 : x_2 = 2x_1, \qquad \ell_2 : x_2 = -\frac{1}{2}x_1 + 50 \qquad \ell_1 \perp \ell_2$

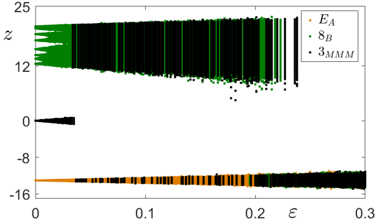




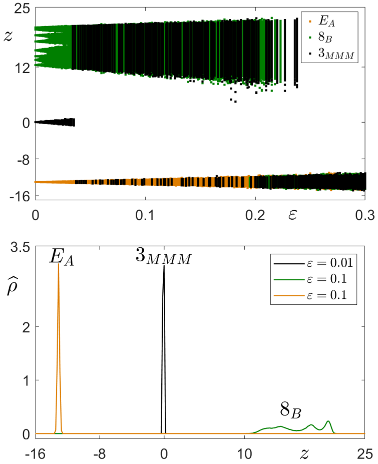
new random variable $z_i = \sqrt{\frac{1}{5}}(2x_{1i} - x_{2i})$

Noise-induced transition $3_{MMM}/C_{MMM} \to E_A$ or $\to 8_B/C_B$

$$D_{12} = 0.00234492,$$
$$D_{21} = 0.00749968$$

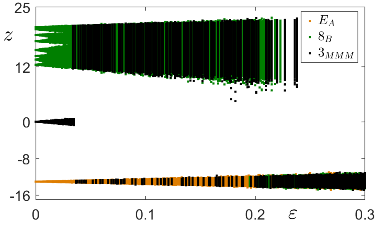


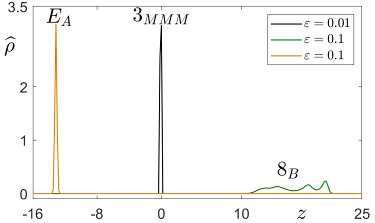
$$D_{12} = 0.00234492,$$
$$D_{21} = 0.00749968$$



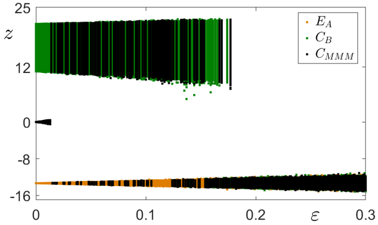
Noise-induced transition $3_{MMM}/C_{MMM} \to E_A$ or $\to 8_B/C_B$

$$D_{12} = 0.00234492,$$
$$D_{21} = 0.00749968$$



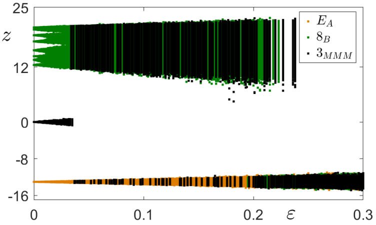


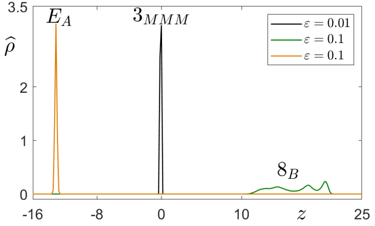
$$D_{12} = 0.002358,$$
$$D_{21} = 0.007552$$



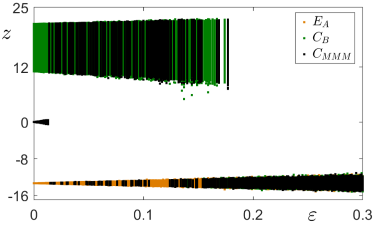
Noise-induced transition $3_{MMM}/C_{MMM} \to E_A$ or $\to 8_B/C_B$

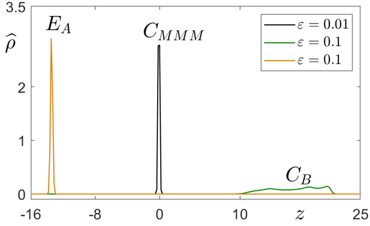
$$D_{12} = 0.00234492, \quad D_{21} = 0.00749968$$





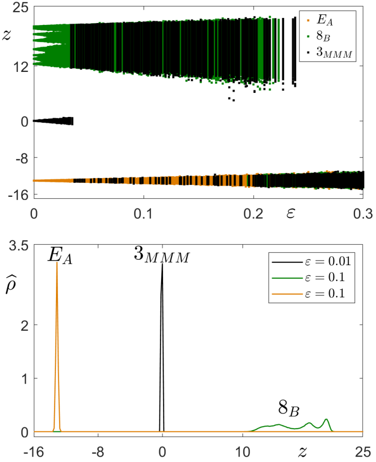
$$D_{12} = 0.002358, \quad D_{21} = 0.007552$$



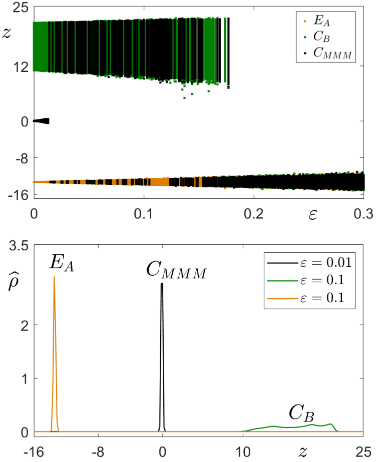


Noise-induced transition $3_{MMM}/C_{MMM} \to E_A$ or $\to 8_B/C_B$

$$D_{12} = 0.00234492,\quad D_{21} = 0.00749968$$



$$D_{12} = 0.002358,\quad D_{21} = 0.007552$$



Stochastic P-bifurcations

Stochastic sensitivity function[2] for an equilibrium

Stochastic equation: case of n-dimensions

$$x_{t+1} = f(x_t, \eta_t), \qquad \eta_t = \varepsilon \xi_t, \tag{3}$$

x_t is an n-dimensional vector, $f(x, \eta)$ is an n-dimensional vector-function,
ε is a scalar parameter denoting the noise intensity,
ξ_t is an m-dimensional uncorrelated random process with parameters
$E\xi_t = 0$, $E\xi_t\xi_t^T = V$, V is a covariance $m \times m$-matrix.

[2] Bashkirtseva I., Ryashko L., Tsvetkov I. (2010) Sensitivity analysis of stochastic equilibria and cycles for discrete dynamic systems. Dynamics of Continuous, Discrete and Impulsive Systems Series A: Mathematical Analysis. 17(4), 501-515.

Stochastic sensitivity function[2] for an equilibrium

Stochastic equation: case of n-dimensions

$$x_{t+1} = f(x_t, \eta_t), \qquad \eta_t = \varepsilon \xi_t, \tag{3}$$

x_t is an n-dimensional vector, $f(x, \eta)$ is an n-dimensional vector-function, ε is a scalar parameter denoting the noise intensity, ξ_t is an m-dimensional uncorrelated random process with parameters $E\xi_t = 0$, $E\xi_t\xi_t^T = V$, V is a covariance $m \times m$-matrix.

Let an exponentially stable equilibrium $\bar{x}$ be an attractor of deterministic model (3) ($\varepsilon = 0$).

The matrix W defining the stochastic sensitivity matrix of equilibrium $\bar{x}$ is a unique solution of following equation:

$$W = FWF^\top + G.$$

Here $F = f'_x(\bar{x}, 0)$, $G = \sigma V \sigma^\top$, $\sigma = f'_\eta(\bar{x}, 0)$.

[2] Bashkirtseva I., Ryashko L., Tsvetkov I. (2010) Sensitivity analysis of stochastic equilibria and cycles for discrete dynamic systems. Dynamics of Continuous, Discrete and Impulsive Systems Series A: Mathematical Analysis. 17(4), 501-515.

Stochastic sensitivity function for a k-cycle

Let $\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k$ be an exponentially stable k-cycle of deterministic system (3) with $\varepsilon = 0$: $\overline{x}_{t+1} = f(\overline{x}_t, 0)$ $(t = 1, 2, ...k-1)$, $\overline{x}_1 = f(\overline{x}_k, 0)$. It holds $\rho(F_k \cdot ... \cdot F_2 \cdot F_1) < 1$.

Stochastic sensitivity function for a k-cycle

Let $\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k$ be an exponentially stable k-cycle of deterministic system (3) with $\varepsilon = 0$: $\overline{x}_{t+1} = f(\overline{x}_t, 0)$ $(t = 1, 2, ...k-1)$, $\overline{x}_1 = f(\overline{x}_k, 0)$. It holds $\rho(F_k \cdot ... \cdot F_2 \cdot F_1) < 1$.

System for k-cycle $\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k$ has a unique stable k-periodic solution W_t:

$$W_{t+1} = F_t W_t F_t^\top + G_t. \tag{4}$$

The set of matrices $\{W_1, ..., W_k\}$ defines the stochastic sensitivity of the elements of the cycle. Here, the element W_1 is a solution of the equation:

$$W_1 = B W_1 B^\top + Q,$$

$$B = F_k \cdot ... \cdot F_2 \cdot F_1,$$

$$Q = G_k + F_k G_{k-1} F_k^\top + ... + F_k \cdot ... \cdot F_2 G_1 F_2^\top \cdot ... \cdot F_k^\top.$$

Other elements $W_2, ..., W_k$ can be found from the equation (4) recurrently.

Here $F_t = f'_x(\overline{x}_t, 0)$, $G_t = \sigma_t V \sigma_t^\top$, $\sigma_t = f'_\eta(\overline{x}_t, 0)$.

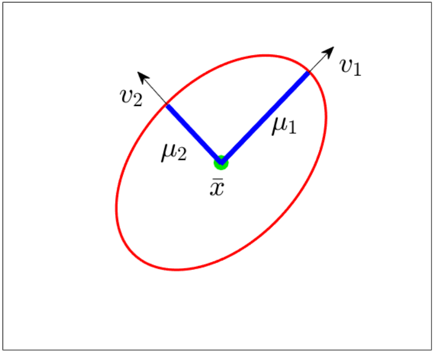
2D: Confidence domain for equilibrium or k-cycle

Confidence ellipse for equilibrium or element of k-cycle:

$$(x - \overline{x}, W^{-1}(x - \overline{x})) = 2\varepsilon q^2$$

$$q^2 = -\ln(1 - \mathcal{P}), \mathcal{P} \text{ is fiducial probability,}$$

μ_i, v_i are eigenvalues and corresponding eigenvectors of W.



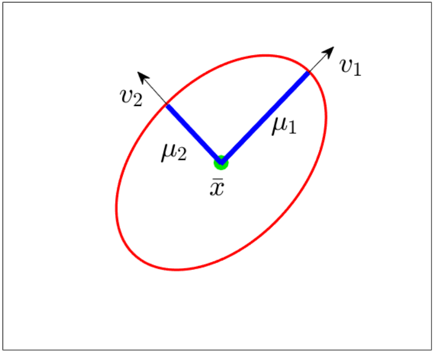
2D: Confidence domain for equilibrium or k-cycle

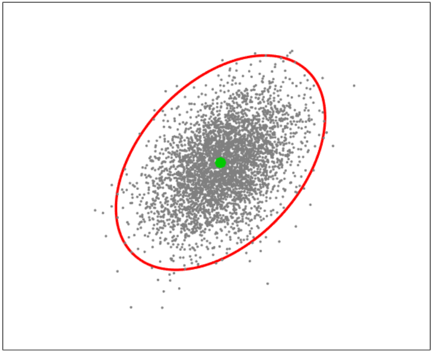
Confidence ellipse for equilibrium or element of k-cycle:

$$(x - \overline{x}, W^{-1}(x - \overline{x})) = 2\varepsilon q^2$$

$$q^2 = -\ln(1 - \mathcal{P}), \mathcal{P} \text{ is fiducial probability,}$$

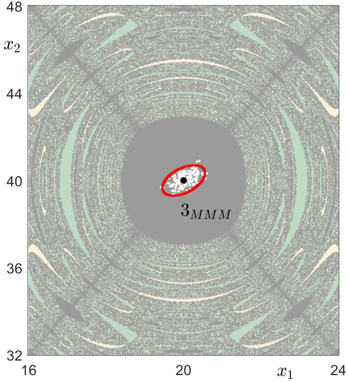
μ_i, v_i are eigenvalues and corresponding eigenvectors of W.





Method of confidence domain for cycle 3_{MMM}

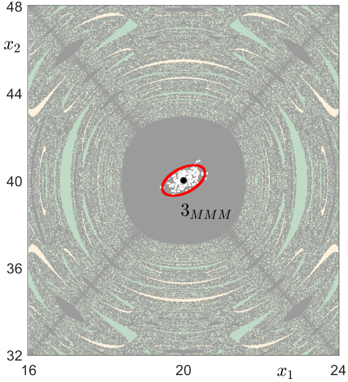
$$D_{12} = 0.00234492,\ D_{21} = 0.00749968$$



$\varepsilon = 0.03$

Method of confidence domain for cycle 3_{MMM}

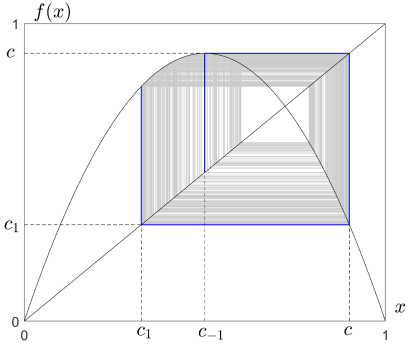
$$D_{12} = 0.00234492, \; D_{21} = 0.00749968$$



$\varepsilon = 0.03$ $\qquad$ $\varepsilon = 0.1$

Let's consider the one-band chaotic attractor $\mathcal{A}$.
Let the function $f(x,0)$ have a unique maximum at the point c_{-1}:

$$f'_x(c_{-1},0)=0, \qquad f(c_{-1},0)=c, \qquad f(c,0)=c_1.$$



[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

1D: Stochastic sensitivity of a chaotic attractor[3]

Let's consider the one-band chaotic attractor $\mathcal{A}$.

Let the function $f(x, 0)$ have a unique maximum at the point c_{-1}:

$$f'_x(c_{-1}, 0) = 0, \qquad f(c_{-1}, 0) = c, \qquad f(c, 0) = c_1.$$

[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

1D: Stochastic sensitivity of a chaotic attractor[3]

Let's consider the one-band chaotic attractor $\mathcal{A}$.
Let the function $f(x,0)$ have a unique maximum at the point c_{-1}:

$$f'_x(c_{-1},0)=0, \qquad f(c_{-1},0)=c, \qquad f(c,0)=c_1.$$

Stochastic sensitivity of the borders c and c_1 of the chaotic attractor is defined as:

$$w(c_1)=[f'_x(f(c,0),0)]^2\, s(c_{-1})+s(f(c_{-1},0)),$$
$$w(c)=s(c_{-1}).$$

[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

1D: Stochastic sensitivity of a chaotic attractor[3]

Let's consider the one-band chaotic attractor $\mathcal{A}$.
Let the function $f(x,0)$ have a unique maximum at the point c_{-1}:

$$f'_x(c_{-1},0)=0, \qquad f(c_{-1},0)=c, \qquad f(c,0)=c_1.$$

Stochastic sensitivity of the borders c and c_1 of the chaotic attractor is defined as:

$$\begin{aligned} w(c_1) &= [f'_x(f(c,0),0)]^2\, s(c_{-1}) + s(f(c_{-1},0)),\\ w(c) &= s(c_{-1}). \end{aligned}$$

Confidence interval

Based on stochastic sensitivity confidence interval around chaotic attractor can be constructed:

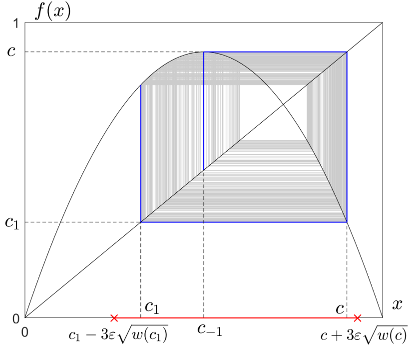
$$(c_1 - 3\varepsilon\sqrt{w(c_1)},\, c + 3\varepsilon\sqrt{w(c)})$$

[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

1D: Stochastic sensitivity of a chaotic attractor[3]

Stochastic sensitivity of the one-band chaotic attractor $\mathcal{A}$:

$$w(c_1) = [f_x'(f(c,0),0)]^2\, s(c_{-1}) + s(f(c_{-1},0)),$$
$$w(c) = s(c_{-1}).$$



[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

Method of confidence domains for chaotic attractor C_{MMM}

$$f : x \mapsto f(x) = 10(\alpha_1 + 4D_{12})x(40 - x)$$

SSF and CD for 3-band chaotic attractor[3]

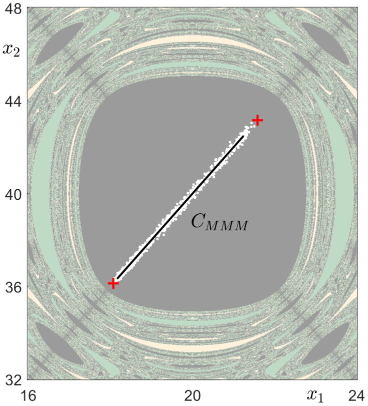
[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

Method of confidence domains for chaotic attractor C_{MMM}

$$f : x \mapsto f(x) = 10(\alpha_1 + 4D_{12})x(40 - x)$$

SSF and CD for 3-band chaotic attractor[3]

$$D_{12} = 0.002358,\ D_{21} = 0.007552$$



$\varepsilon = 0.005$

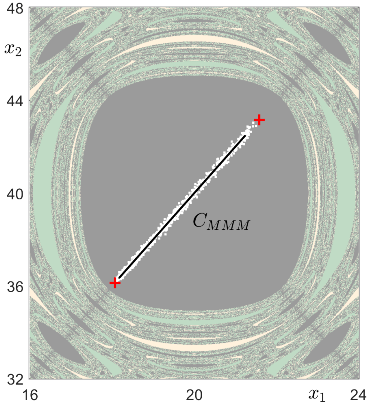
[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

Method of confidence domains for chaotic attractor C_{MMM}

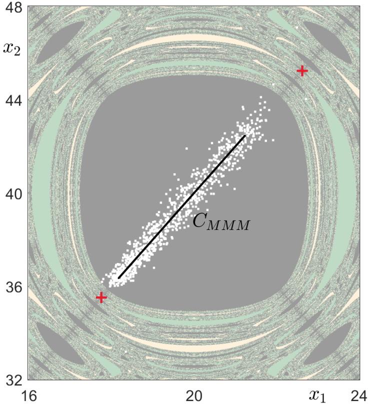
$$f : x \mapsto f(x) = 10(\alpha_1 + 4D_{12})x(40 - x)$$

SSF and CD for 3-band chaotic attractor[3]

$$D_{12} = 0.002358, \; D_{21} = 0.007552$$



$\varepsilon = 0.005$



$\varepsilon = 0.02$

[3] Bashkirtseva I. and Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.

References I

Stochastic sensitivity function technique (SSF)

First steps in developing the stochastic sensitivity:

- Ryashko L.B. (1979). Linear filter in the stabilization problem for linear stochastic-systems with incomplete information. Automation and Remote Control, 40(7), 1010-1018.
- Ryashko L.B., Milshtein G.N. (1984). Estimation in controlled stochastic-systems with multiplicative noise. Automation and Remote Control, 45(6), 759-765.

SSF of a fixed point and k-cycle:

- Bashkirtseva I., Ryashko L., Tsvetkov I. (2010) Sensitivity analysis of stochastic equilibria and cycles for discrete dynamic systems. Dynamics of Continuous, Discrete and Impulsive Systems Series A: Mathematical Analysis. 17(4), 501-515.

SSF of a closed invariant curve:

- Bashkirtseva I., Ryashko L. (2014) Stochastic sensitivity analysis of the attractors for the randomly forced Ricker model with delay. Physics Letters A. 378, 3600-3606.

SSF of a chaotic attractor:

- Bashkirtseva I., Ryashko L. (2017). Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. Physics Letters A, 381(37):3203-3210.
- Bashkirtseva I., Ryashko L. (2019). Stochastic sensitivity analysis of chaotic attractors in 2D non-invertible maps. Chaos, Solitons and Fractals, 126, 78-84.

References II

Previous works:

- Gaertner, W., Jungeilges, J. (1988) A non-linear model of interdependent consumer behaviour. Economics Letters, 27 (2), 45-150. https://doi.org/10.1016/0165-1765(88)90087-0
- Gaertner, W., Jungeilges, J. (1993) 'Spindles' and coexisting attractors in a dynamic model of interdependent consumer behavior. A note. Journal of Economic Behavior and Organization, 21 (2), 223-231. https://doi.org/10.1016/0167-2681(93)90049-U
- Gaertner, W., Jungeilges, J. (1994) A model of interdependent consumer behavior: Nonlinear dynamics in R^2 Eichhorn W. (Ed.), Models and measurement of welfare and inequality, Springer-Verlag, Berlin Heidelberg New York, 940-962
- Ekaterinchuk, E., Jungeilges, J., Ryazanova, T., Sushko, I. (2017). Dynamics of a minimal consumer network with uni-directional influence. Journal of Evolutionary Economics, 27(5), 831-857. https://doi.org/10.1007/s00191-017-0517-5
- Ekaterinchuk, E., Jungeilges, J., Ryazanova, T., Sushko, I. (2018). Dynamics of a minimal consumer network with bi-directional influence. Communications in Nonlinear Science and Numerical Simulation, 58, 107-118. https://doi.org/10.1016/j.cnsns.2017.04.007
- Jungeilges, J., Ryazanova, T., Mitrofanova, A., Popova, I. (2018). Sensitivity analysis of consumption cycles. Chaos, 28(5), [055905]. https://doi.org/10.1063/1.5024033
- Jungeilges, J., Ryazanova, T. (2019). Transitions in consumption behaviors in a peer-driven stochastic consumer network. Chaos, Solitons and Fractals, 128, 144-154. https://doi.org/10.1016/j.chaos.2019.07.042
- Jungeilges, J., Nilssen, T. K., Perevalova, T., Satov, A. (2021). Transitions between metastable long-run consumption behaviors in a stochastic peer-driven consumer network. Discrete and Continuous Dynamical Systems - Series B, 26(11), 5849-5871. https://doi.org/10.3934/DCDSB.2021232
- Jungeilges, J., Pavletsov, M., Perevalova, T. (2022). Noise-induced behavioral change driven by transient chaos. Chaos, Solitons and Fractals, 158, [112069]. https://doi.org/10.1016/j.chaos.2022.112069
- Jungeilges, J., Nilssen, T.K., Pavletsov, M., Perevalova, T. (2025) Consumption Dynamics in Mixed-Income Neighborhoods with Connected Households. Computational Economics, 65, 1051-1082. https://doi.org/10.1007/s10614-024-10774-3
- Jungeilges, J., Nilssen, T.K., Pavletsov, M., Perevalova, T. Noise-induced multistage transitions in a dynamic model of rational consumer choice. Decisions in Economics and Finance. Under Review

The End

Gracias por su atención!

Thank you for your attention!