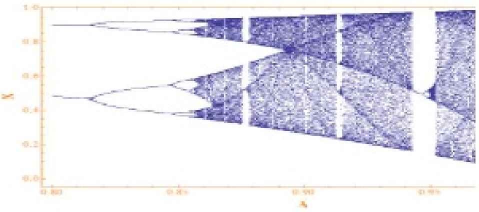


Progress on Difference Equations
International Conference
PODE 2025
Cartagena, 28th-30th May 2025

Progress on Difference Equations 2025

SéNeCa(+)
Agencia de Ciencia y Tecnología
Región de Murcia

Región de Murcia
Consejería de Medio Ambiente, Universidades, Investigación y Mar Menor

Universidad Politécnica de Cartagena

UNIVERSIDAD DE MURCIA

International Society of Difference Equations
ISDE

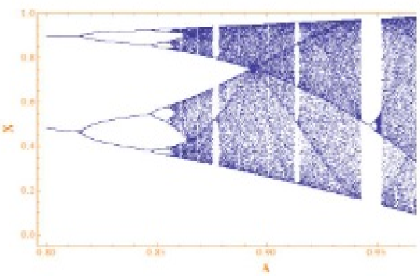
industriales
etsii UPCT

Progress on Difference Equations 2025, PODE25

Cartagena, May, 28-30, 2025

Presentation

Welcome



Progress on Difference Equations
International Conference
PODE 2025
Cartagena, 28th-30th May 2025

- Welcome to *Progress on Difference Equations 2025, PODE25,* which will take place from 28th to 30th of May, in Cartagena, Spain. We hope that you find the conference interesting from scientific and social point of view.

- This is the 14th International Conference which follows those held each year since 2007 on the same topics. After 2017, the workshop PODE alternates each year with the European Conference on Iteration Theory - ECIT and therefore is being held every two years.

- The conference in Cartagena aims to continue the tradition of previous PODE conferences. The first PODE conference took place at Laufen (Germany, 2007), and was followed by the meetings of Laufen (Germany, 2008), Bedlewo (Poland, 2009), Xanthi (Greece, 2010), Dublin (Ireland, 2011), Richmond (Virginia, USA, 2012), Bialystok (Poland, 2013), Izmir (Turkey, 2014), Covilhã (Portugal, 2015), Riga (Latvia, 2016), Urbino (Italy, 2017), Bragança (Portugal, 2019) and Milan (Italy, 2023).

- PODE 2025 is held under the auspices of the International Society for Difference Equations, the Group of Dynamical Systems of Murcia, the Department of Mathematics of the Universidad de Murcia and the Department of Applied Mathematics and Statistics of Universidad Politécnica de Cartagena.

- Cartagena is a small city located in the southeast of Spain, with an excellent natural harbour and a history of more than 2000 years. The conference attendees can enjoy not only the interesting scientific program but also the warm weather, the typical Spanish gastronomy and the nice cultural life of the city.

- The conference aims to be a forum for young and senior researchers to share their work and discuss the latest developments in the areas of difference equations, discrete dynamical systems, functional equations and their applications.

- The activities of the Conference will take place at **Salón de Grados**, which is located at Escuela Técnica Superior de Ingeniía Industrial, at Technical University of Cartagena. In Figures 1 and 2 you can see how to connect Hotels Carlos III and Alfonso XIII with the Conference site. The hotels are very close, so all the paths included in the maps are valid.

Figure 1: Map of Cartagena. Path from Hotel Carlos III and ETSII building, which is the rectangular one with two inner squares.

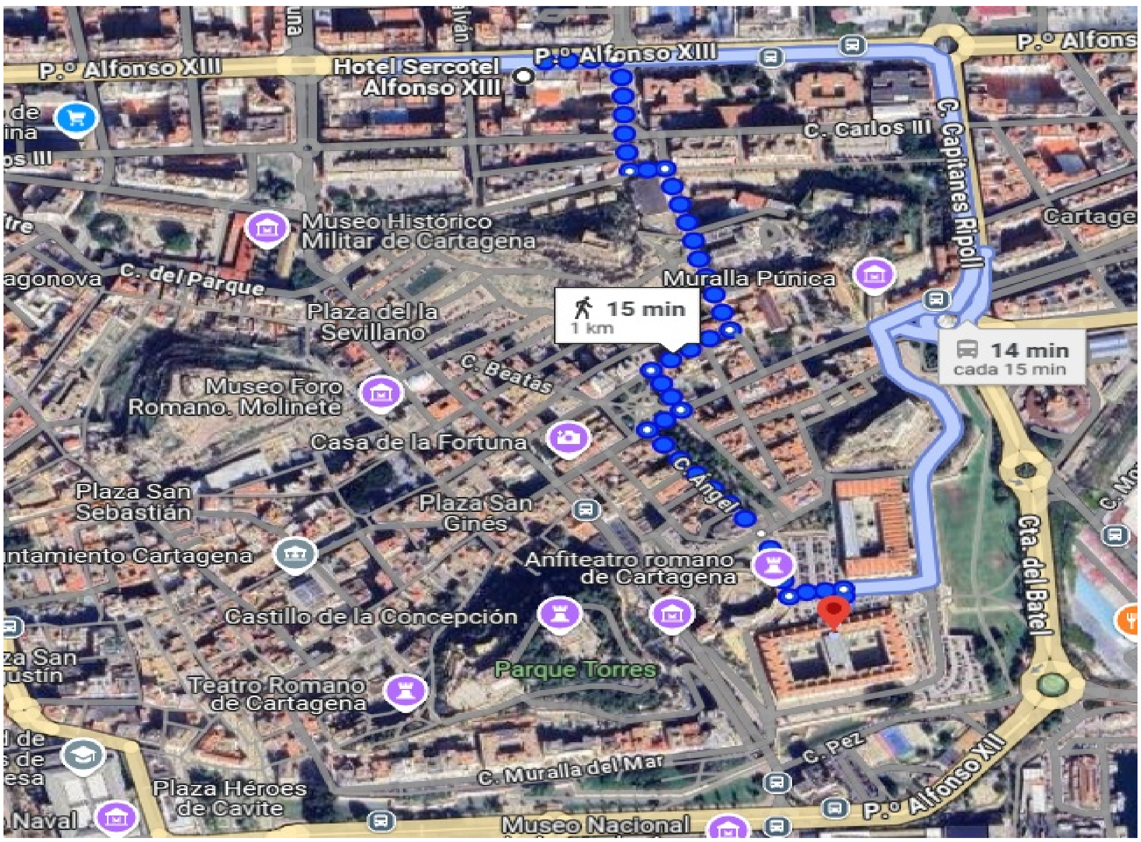


Figure 2: Map of Cartagena. Path from Hotel Alfonso XIII and ETSII building, which is the rectangular one with two inner squares.

Social activities

- During the Conference, we are planning to organise two excursions on Thursday. The first one will be devoted to visiting the *Roman Theatre Museum*. We will visit the museum and go into the Roman Theatre. In doing so, we will follow a path where all the ancient cultures of Cartagena, Carthago, Roman, Bizantine, Muslim, and Christian are met. The second excursion is planned to visit Cartagena Port landscapes, including a small trip on a tourist ship. It is highly recommended to use a small cap during the ship excursion. If you do not wish to take such an excursion, you can wait for us to have something in the "chiringuitos" located in the port.

- The traditional conference dinner will be at Hotel Los Habaneros, a well-known restaurant in the city centre of Cartagena (see Figure 3), and close to the conference building. We hope you enjoy the dinner, which will be on Thursday at 20:30.

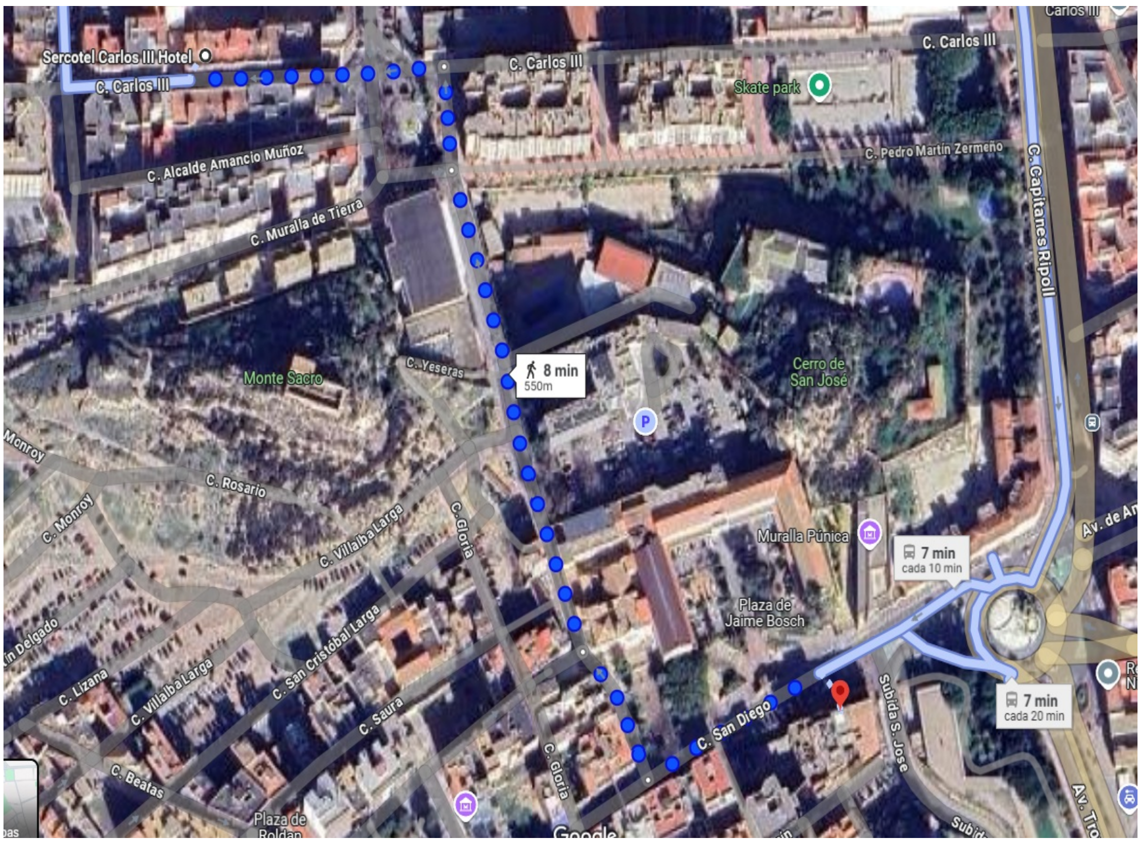


Figure 3: Map of Cartagena. Path from Hotels Carlos III and Los Habaneros. In front of Hotel Los Habaneros you can see the main building of the universitity.

Organized by

- Department of Applied Mathematics and Statistics. Technical University of Cartagena.
- Department of Mathemtics. Murcia University.
- International Society of Difference Equations.

Local Sponsors

- Fundación Séneca.
- Dirección General de Universidades e Investigación de la Consejería de Medio Ambiente, Universidades, Investigación y Mar Menor.
- Escuela Técnica Superior de Ingernía Industrial.
- Universidad Politécnica de Cartagena.

Organizing Committee

- Jose S. Cánovas, Department of Applied Mathematics and Statitistics, Technical University of Cartagena.
- José Ginés Espín, Department of Mathematics, Murcia University.
- Víctor Jiménez López, Department of Mathematics, Murcia University.
- Antonio Linero Bas, Department of Mathematics, Murcia University.
- María Muñoz Guillermo, Department of Applied Mathematics and Statitistics, Technical University of Cartagena.
- Daniel Nieves Roldán, Department of Mathematics, Murcia University.
- María del Carmen Ruiz Abellón, Department of Applied Mathematics and Statitistics, Technical University of Cartagena.
- Gabriel Soler López, Department of Applied Mathematics and Statitistics, Technical University of Cartagena.

Scientific Committee

- Saber Elaydi, Trinity University, USA.
- Laura Gardini, University of Urbino Carlo Bo, Italy.
- Armengol Gasull, Universitat Autónoma de Barcelona, Spain.
- René Lozi, Université Côte d'Azur, France.
- Adina Luminita Sasu, West University of Timisoara, Romania.
- Lubomir Snoha, Matej Bel University, Slovakia.
- Jose S. Cánovas, Technical University of Cartagena, Spain.

Invited Speakers

- Saber Elaydi, Trinity University, USA.
- David Juher, Universitat de Girona, Spain.
- Senada Kalabusic, University of Sarajevo, Bosnia and Herzegovina.
- Víctor Mañosa, Universitat Politécnica de Catalunya, Spain.
- Davide Radi, Universit Cattolica del Sacro Cuore, Italy.

Timetable

Progress on Difference Equations 2025

Scientific Program

Wednesday, 28 of May

9:00	Opening Ceremony
9:15	Saber Elaydi Evolutionary Discrete-Time Models in Ecology and Epidemiology **Chair:** Laura Gardini
10:00	Coffee break
10:30	Contributed talks. Session 1.
13:00	Lunch Time
16:15	Senada Kalabusic Difference equations in mathematical modeling: Exploring the Rosenzweig-MacArthur predator-prey model (stability, bifurcations, permanence) **Chair:** Saber Elaydi
17:00	Coffee break
17:30	Contributed talks. Session 2.

Contributed talks. Session 1. Chair: Francisco Balibrea

10:30	Azmy S. Ackleh How predator evolution to lethal or sublethal toxicant effects alters the dynamics of a discrete-time predator-prey system
11:00	Fangfang Liao Non-constant periodic solutions of the Ricker model with periodic parameters
11:30	Juan Segura Discrete population models and dispersal: Part I
12:00	Daniel Franco Discrete population models and dispersal: Part II
12:30	Naida Mujić Codimension-one and -two bifurcations and stability of certain second order rational difference equation with arbitrary parameters

Contributed talks. Session 2. Chair: Iryna Sushko

17:30	Tatyana Perevalova Noise-induced de-synchronization in dynamic stochastic consumption model
18:00	Mauro Maria Baldi Exploring the κ-logistic Growth Model: Dynamics and Insights
18:30	Mourad Azioune / René Lozi Chaos Control and Mixed Mode Oscillations in a Bertrand Duopoly Model with Quadratic Costs
19:00	Jochen Jungeilges Sensitivity analysis for attractors of a stochastic piecewise ARMA(2,2)-type asset-price model
19:30	Houssem Eddine Rezgui The dynamic of q-deformed logistic maps

Progress on Difference Equations 2025

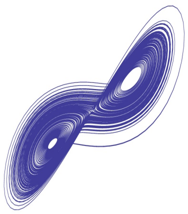
Scientific Program

Thursday 29 of May

9:15	David Juher / Lluís Alsedá Volume entropy of surface groups: a dynamical approach **Chair:** Antonio Linero
10:00	Coffee break
10:30	Contributed talks. Session 3.
13:00	Lunch Time
16:00	Excursion. Visit to Roman Theatre Musseum and ship
20:30	Conference Dinner

Contributed talks. Session 3. Chair: Senada Kalabusic

10:30	David Rojas Structure of zero entropy patterns of trees
11:00	Jakub Hesoun Connecting stationary fronts of Nagumo lattice differential equation with a functional equation
11:30	Ewa Girejko On numerical analysis of car-following models with discrete fractional operators
12:00	Dorota Mozyrska Logistic fractional variable order discrete-time equation with distributed delays
12:30	Amira Mchaalia On totally periodic ω-limit sets for monotone maps on regular curves



Progress on Difference Equations 2025

Scientific Program

Friday 30 of May

9:15	Víctor Mañosa Dynamics on invariant graphs in planar continuous piecewise linear maps **Chair:** Armengol Gasull
10:00	Coffee break & Poster Session
10:30	Contributed talks. Session 4.
13:00	Lunch
16:15	Davide Radi Abundance of weird quasiperiodic attractors in piecewise linear discontinuous maps **Chair:** René Lozi
17:00	Coffee break
17:30	Contributed talks. Session 5.
20:00	Closing Ceremony

Contributed talks. Session 4. Chair: Davide Radi

10:30	Armengol Gasull On the dynamics of a family of planar discontinuous piecewise linear maps
11:00	Daniel Nieves Roldán On the dynamics of a family of max-type difference equations
11:30	José Ginés Espín Buendía Attracting fixed points for the Buchner-Żebrowski equation: the role of negative Schwarzian derivative
12:00	Aymen Daghar Topological sequence entropy of tree maps
12:30	Iryna Sushko On the emergence and properties of weird quasiperiodic attractors

Contributed talks. Session 5. Chair: Azmy S. Ackleh

17:30	Laura Gardini Weird quasiperiodic attractors in two-dimensional PWL and PWS triangular maps
18:00	Francisco Balibrea Difference equations related to Thue-Morse, Fibonnaci and Rudin-Shapiro sequences
18:30	Peter Raith Multifractal dimensions of invariant subsets for piecewise monotonic maps
19:00	Eddy Kwessi Wave Propagation and Global Dynamics of Invasion via Lattice Difference Equations
19:30	Dorsaf Bazzaa Dynamics of Population Growth: Stability Analysis of the Ricker Model

Poster Session.

10:00	Khadija Ben Rejeb Existence proof of a chaotic attractor for the Lozi mappings
10:00	María Muñoz-Guillermo Weird quasiperiodic attractors in two-dimensional PWL and PWS triangular maps
10:00	Gabriel Soler López On the emergence and properties of weird quasiperiodic attractors

Plenary Talks

Evolutionary Discrete-Time Models in Ecology and Epidemiology

Saber Elaydi*

This talk focuses on the application of evolutionary discrete-time models in the study of population dynamics, particularly in the fields of ecology and epidemiology. These models provide essential insights into the interactions between ecological processes and evolutionary dynamics, especially in relation to disease transmission, adaptation, and resistance in populations.

We begin by reviewing discrete-time models and their incorporation of evolutionary processes such as selection, mutation, and migration. These models are especially useful in studying the spread of diseases and the evolution of pathogen resistance, providing a mathematical framework for understanding the dynamics of evolving populations.

The global dynamics of studies systems will be analyzed using theorems on mixed monotone maps, Liapunov functions, and other techniques. Recent work (Elaydi et al., 2025; Cushing et al., 2023) will be utilized, highlighting how bifurcation theory and stability analysis are applied to model the evolution of resistance in diseases like malaria and tuberculosis. The talk emphasizes the role of bifurcation analysis in identifying critical thresholds in population dynamics, where small changes in parameters can lead to significant shifts, such as the emergence of new disease strains or population collapse.

References

1. S. N. Elaydi and J. M. Cushing, Discrete Mathematical Models in Population Biology: Ecological, Evolutionary, and Epidemic, *Springer.* (2025)

2. Cushing, J. M., Henson, S. M., & Hayward, J. L, Dynamics,Dynamics of Marine Vertebrates: Behavioral, Population, and Evolutionary Models, *Springer.* (2023).

* **e–mail:** *selaydi@trinity.edu*

Volume entropy of surface groups: a dynamical approach

Lluís Alsedá*, David Juher, Jérôme Los, Francesc Mañosas

In this talk we solve [1,2] a purely algebraic problem using dynamical systems tools, in particular the theory of kneading invariants of Milnor-Thurston. Consider the fundamental group G of a compact surface. Let $P = \langle X|R\rangle$ be any presentation of G, where X is a set of generators and R is a set of relations (words in the alphabet X that are equivalent to the identity element). The presentation P is called geometric if the associated Cayley graph is planar. The volume entropy of G with respect to P is an important quantity in Geometric Group Theory. It is defined as the exponential growth rate, as m goes to infinity, of the number of vertices in the Cayley graph at distance m from the identity vertex. In this setting, we will construct a dynamical system defined by a piecewise continuous and monotone map of the circle, whose topological entropy is computable and coincides with the volume entropy. The algorithm, which is explicit and depends only on the set of relations R, has been implemented in Maple and Maxima languages.

References

1. Ll. Alsedà, D. Juher, J. Los, F. Mañosas, On families of Bowen-Series-like maps for surface groups, *Regul. Chaotic Dyn.*, 28(4-5) (2023), 659–667.

2. Ll. Alsedà, D. Juher, J. Los, F. Mañosas, Entropy stability and Milnor-Thurston invariants for Bowen-Series-like maps, *Preprint* (2024).

* **e–mail:** *david.juher@udg.edu*

Difference equations in mathematical modeling: Exploring the Rosenzweig-MacArthur predator-prey model (stability, bifurcations, permanence)

Senada Kalabusic*, Emin Beso, Esmir Pilav

This presentation explores the Rosenzweig-MacArthur predator-prey model, an important concept in ecology and mathematical biology. The model is defined by difference equations derived from Euler discretization and features a general functional response for the predator.

Our main focus is identifying an open, positively invariant set of the system. This set is more than just a theoretical construct; it is crucial to ensure that all subsequent analyses are biologically meaningful, enhancing the model's applicability to real-world ecological systems.

The presentation is divided into two parts. In the first part, we will briefly introduce the Rosenzweig-MacArthur predator-prey model. The second part will examine the closed positive invariant set and the complex dynamics of the system within that set. We will also demonstrate that, despite the observed dynamics, the system remains permanent within this set, highlighting the model's complexity.

References

1. E. C. Balreira, S. Elaydi, S., R. Luis, *Local stability implies global stability for the planar Ricker competition model*, Discrete and Continuous Dynamical Systems - Series B 19(2) (2014), pp. 323-351. http://doi.org/10.3934/dcdsb.2014.19.32.

2. E. Beso, S.K., E. Pilav, *Dynamics of the discrete-time Rosenzweig-MacArthur predator-prey system in the closed positively invariant set*, Computational and Applied Mathematics 42, 341 (2023). https://doi.org/10.1007/s40314-023-02481-w.

3. G. F. Gause, *The struggle for existence. Williams and Wilkins*, Baltimore, 1934.

4. Gause G. F., Smaragdova N.P., Witt A.A., *Further studies of the interaction between predator and prey*, J Animal Ecol 5 (1936), pp. 118.

5. H. I. Freedman and Joseph W.-H. So, *Persistence in Discrete Semidynamical Systems*, SIAM Journal on Mathematical Analysis 20(1989). DOI:10.1137/0520062.

6. H. I. Freedman and P. Waltman, *Persistence in models of three interacting predator-prey populations*, Mathematical Biosciences, 68(2)(1984), pp. 213231.

7. Gregor F Fussmann, Bernd Blasius, *Community response to enrichment is highly sensitive to model structure*, Biol. Lett. 1 (2005), pp. 9-12. doi: 10.1098/rsbl.2004.0246.

8. J. Hofbauer, V. Hutson, and W. Jansen, *Coexistence for systems governed by difference equations of Lotka-Volterra type*, J. Math. Biol. (25) (1987), pp. 553-570.

9. R. Kon and Y. Takeuchi, *Permanence of host-parasitoid system*, Nonlinear Analysis 47 (2001), pp. 1383-1393.

10. Vlastimil Krivan, *On the Gause predator-prey model with a refuge: a fresh look at the history,* Journal of Theoretical Biology, Volume 274, Issue 1 (2011), pp. 67-73.

11. M.R. Myerscough, M.J. Darwen, W.L. Hogarth, *Stability, persistence and structural stability in a classical predator-prey model*, Ecol. Modeling 89 (1980), pp. 3142.

12. S.E. Newhouse, *The abundance of wild hyperbolic sets and non-smooth stable sets for diffeomorphisms*, Publications Mathmatiques de lIHS 50(1) (1979), pp. 101151. Doi:10.1007/bf02684771 10.1007/bf02684771

13. Alexandre A. Rodrigues, *Abundance of Strange Attractors Near an Attracting Periodically Perturbed Network*, SIAM Journal on Applied Dynamical Systems (20) (2021). Doi:(2021)10.1137/20M1335510

14. M.L. Rosenzweig, R.H. MacArthur, *Graphical representation and stability conditions of predator-prey interactions*, Am. Nat. 97 (1963), pp. 209-223.

15. M.L. Rosenzweig, *Paradox of enrichment: destabilization of exploitation ecosystems in ecological time*, Science 171 (1971), pp. 385387.

16. S.J. Schreiber, *Criteria for Cr Robust Permanence*, Journal of Dierential Equations 162 (2000), pp. 400-426.

17. S. J. Schreiber,*Chaos and population disappearances in simple ecological models*, J. Math. Biol. 42 (2001), pp. 239260.

18. Gunog Seo, Gail S. K. Wolkowicz, *Sensitivity of the dynamics of the general RosenzweigMacArthur model to the mathematical form of the functional response: a bifurcation theory approach*, J. Math. Biol.https://doi.org/10.1007/s00285-017-1201-y 2018.

19. Hal L. Smith, The Rosenzweig-MacArthur predator-prey model, math.la.asu.edu

20. Q. Wang, L.S. Young, *Dynamical prole of a class of rank-one attractors*, Ergod. Th. & Dynam. Sys.(33) (2013), pp. 12211264 Cambridge University Press.

21. H. Whitney, *On singularities of mappings of euclidean spaces. mappings of the plane into the plane*, Annals of Mathematics 62(3) (1955), pp. 374410.

22. Stephen Willard, General Topology, Dover Publication, 2004.

* **e–mail:** *senadak@pmf.unsa.ba*

Dynamics on invariant graphs in planar continuous piecewise linear maps

Víctor Mañosa*

From the foundational works of Lozi and Devaney [2, 3], it is well-known that planar continuous piecewise linear maps can exhibit complex behaviors, particularly chaotic dynamics. In our recent works [5,6], we identified what we believe to be a novel phenomenon in continuous piecewise linear maps: the existence of compact one-dimensional graphs that capture the global dynamics in the plane. Within these graphs, chaotic dynamics emerge for certain parameter values, giving rise to an intermediate dynamical regime between regular behavior and full-plane chaos.

Although this behavior is not exclusive to this family, we detected it in the family defined by

$$F(x,y) = (|x| - y + a, x - |y| + b)\,,$$

where $(a,b) \in \mathbb{R}^2$, which has been analyzed in previous studies where this behavior does not appear, such as [1, 4, 7, 8].

In this talk, I will present the main results from [5], focusing on the parameter values for which these graphs exist. Through specific examples, I will explain in detail how these invariant graphs are formed and the types of dynamics they exhibit. In particular, I will outline the arguments used to compute or bound the topological entropy for the maps restricted to these graphs.

The results I will present are part of joint work with Anna Cima, Armengol Gasull, and Francesc Mañosas from the Universitat Autònoma de Barcelona.

References

1. B. Aiewcharoen, R. Boonklurb, N. Konglawan, Global and local behavior of the system of piecewise linear difference equations $x_{n+1} = |x_n| - y_n - b$ and $y_{n+1} = x_n - |y_n| + 1$ where $b \leq 4$. *Mathematics*, **9** (2021), 1390-27pp.

2. R. L. Devaney, A piecewise linear model for the zones of instability of an area-preserving map, *Physica D* **10** (1984), 387-393.

3. R. Lozi, Un attracteur étrange (?) du type attracteur de Hénon, *J. Phys. Colloques*, **39** (1978), C5–9–C5–10.

4. I. Bula, A. Sīle, About a system of piecewise linear difference equations with many periodic solutions, in S. Olaru et al. (eds.). Difference Equations, Discrete Dynamical Systems and Applications, Springer Proceedings in Mathematics & Statistics 444, 29–50. Springer, New York, 2024.

5. A. Cima, A. Gasull, V. Mañosa, F. Mañosas, Invariant graphs and dynamics of a family of continuous piecewise linear planar maps. *Qualitative Theory of Dynamical Systems* **24**:70 (2025), 103 pp.

6. A. Cima, A. Gasull, V. Mañosa, F. Mañosas, Further results for a family of continuous piecewise linear planar maps, arXiv:2503.02411 [math.DS]

7. W. Tikjha, Y. Lenbury, E. Lapierre, Equilibriums and periodic solutions of related systems of piecewise linear difference equations, *Int. J. Math. Comput. Sim.*, (2013), 323–335.

8. W. Tikjha, E. Lapierre, T. Sitthiwirattham. The stable equilibrium of a system of piecewise linear difference equations, *Adv. Difference Equ.*, (2017), Paper No. 67, 10pp.

* **e–mail:** *victor.manosa@upc.edu*

Abundance of weird quasiperiodic attractors in piecewise linear discontinuous maps

Davide Radi* Laura Gardini, Noemi Schmitt, Iryna Sushko, Frank Westerhoff

In this work, we consider a class of n-dimensional, $n \geq 2$, piecewise linear discontinuous maps that can exhibit a new type of attractor, called a weird quasiperiodic attractor (discovered for the first time in [1] where a financial market model is considered, see also [4]). While the dynamics associated with these attractors may appear chaotic, we prove that a chaotic attractor cannot occur. The considered class of n-dimensional maps allows for any finite number of partitions, separated by various types of discontinuity sets. The key characteristic, beyond discontinuity, is that all functions defining the map have the same real fixed point. These maps cannot have hyperbolic cycles other than the fixed point itself. We consider the two-dimensional case in detail. We prove that in nongeneric cases, the restriction, or the first return, of the map to a segment of straight line issuing from the fixed point is reducible to a piecewise linear circle map (see, e.g., [3]). The generic attractor, different from the fixed point, is a weird quasiperiodic attractor, which may coexist with other attractors or attracting sets. We illustrate the existence of these attractors through numerous examples, using functions with different types of Jacobian matrices, as well as with different types of discontinuity sets. In some cases, we describe possible mechanisms leading to the appearance of these attractors. We also give examples in the three-dimensional space. Several properties of this new type of attractor remain open for further investigation, see also [2].

References

1. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff,*On the limits of informationally efficient stock markets: New insights from a chartist-fundamentalist model.* Submitted for publication. (2025) https://arxiv.org/abs/2410.21198.

2. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, *On the emergence and properties of weird quasiperiodic attractors,* Submitted for publication. (2025) https://arxiv.org/pdf/2503.11264.

3. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, *Dynamics of 1D discontinuous maps with multiple partitions and linear functions having the same fixed point. An application to financial market modeling*, Working Paper, University of Bamberg (2025) (DOI: 10.48550/arXiv.2503.20449, https://arxiv.org/abs/2503.20449).

4. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, *How risk aversion may shape the dynamics of stock markets: a chartist-fundamentalist approach,* Working Paper, University of Bamberg (2025).

* **e–mail:** *davide.radi@unicatt.it*

Contributed talks and Posters

How predator evolution to lethal or sublethal toxicant effects alters the dynamics of a discrete-time predator-prey system

Azmy S. Ackleh*, Neerob Basak, Amy Veprauskas

We extend the predator-prey model developed in [1] by incorporating the evolution of a predators resistance to toxicant effects. Specifically, we examine three scenarios: (1) lethal effects, where the toxicant influences predator survival; (2) sublethal effects, where the toxicant affects predator fecundity; and (3) mixed effects, where both survival and fecundity are impacted. Assuming a trade-off between toxicant resistance and prey capture ability, we model the evolutionary dynamics of the predator under sustained toxicant exposure. For the first two cases, we derive conditions for the existence and stability of equilibria, as well as criteria for system persistence. All three cases are further explored through numerical simulations to gain deeper insight into the role of predator evolution on system dynamics. Our results show that the evolution of resistance can enable predator persistence under conditions that would otherwise lead to extinction. However, in the case of lethal effects, we find that evolution can sometimes give rise to multiple stable boundary equilibria. In such instances, evolution in response to toxicant exposure may paradoxically lead to predator extinction, whereas in the absence of resistance evolution, the predator population would persist.

References

1. Ackleh, A. S., Hossain, M. I., Veprauskas, A., and Zhang, A. (2019). Persistence and stability analysis of discrete-time predator-prey models: a study of population and evolutionary dynamics. Journal of Difference Equations and Applications, 25(11):1568-1603.

* **e–mail:** *azmy.ackleh@louisiana.edu*

Chaos Control and Mixed Mode Oscillations in a Bertrand Duopoly Model with Quadratic Costs

Mourad Azioune*, Mohammed Salah Abdelouahab, René Lozi

In this paper, we examine the emergence of complex dynamics, including chaos, bifurcations, and mixed mode oscillations, in a Bertrand duopoly model where firms compete with homogeneous expectations and quadratic cost functions. By analyzing the nonlinear behavior of the system, we identify conditions leading to instability, flip bifurcations, and chaotic fluctuations in market prices. We explore how flip bifurcations play a crucial role in the transition to chaos, leading to unpredictable market behavior. To address these irregularities, we employ a state feedback control approach designed to suppress chaos and restore stable equilibrium. Our results highlight the interplay between competition, cost structures, bifurcation mechanisms, and dynamic stability, offering insights into the application of chaos control methods in economic modeling.

Keywords: Bertrand duopoly, chaos control, bifurcation, mixed mode oscillations, nonlinear dynamics.

References

1. Azioune, M., & Abdelouahab, M. S. (2025). Controlling chaos and mixed mode oscillations in a Bertrand duopoly game with homogeneous expectations and quadratic cost functions. *Mathematics and Computers in Simulation*, 233, 553-566.

2. Amira, R., Abdelouahab, M. S., Touafek, N., Mesmouli, M. B., Zaidi, H. N., & Hassan, T. S. (2024). Nonlinear dynamics in a remanufacturing duopoly game: spectral entropy analysis and chaos control. *AIMS Mathematics*, 9(3), 7711-7727.

* **e–mail:** *m.azioune@centre-univ-mila.dz*

Exploring the κ-logistic Growth Model: Dynamics and Insights

Mauro Maria Baldi*, Cristiana Mammana, Elisabetta Michetti

Economic growth, central in Macroeconomics, has been analyzed through various models, starting with Ramsey [1] and Solow-Swan [2]. Böhm and Kaas [3] proposed a model analyzing capital per capita in discrete time, introducing differential savings between workers and shareholders. They ensured a unique steady state by using the weak Inada conditions.

Subsequent models relaxed these assumptions [4,5,6,7].

This paper introduces the κ-logistic function, a generalization of the logistic function, based on the κ-exponential function ($\exp_\kappa$) by Kaniadakis [8]. The κ-logistic function accounts for rare events affecting economic growth, such as natural disasters or epidemics. It provides more flexibility, modeling both concave and convex-concave production functions, and allows varying upper bounds for production, adapting to economies at different stages of development.

The model, analyzed through analytical and numerical methods, exhibits behaviors such as concave, non-concave, and bimodal dynamics. It may display multistability, where the economy's equilibrium depends on initial conditions. In low-development economies or those frequently impacted by rare events, poverty traps can emerge. In contrast, developed economies can achieve growth, even with rare shocks.

In conclusion, the κ-logistic model offers a flexible approach to studying economic dynamics, incorporating both typical growth behavior and the effects of rare, extreme events.

Acknowledgement

Mauro Maria Baldi and Elisabetta Michetti have been funded by the European Union - NextGenerationEU under the Italian Ministry of University and Research (MUR) National Innovation Ecosystem grant ECS00000041 - VITALITY - CUP D83C22000710005.

References

1. F. P. Ramsey. A Mathematical Theory of Saving. *The Economic Journal*, 38(152):543–559, 1928.
2. R. M. Solow. A Contribution to the Theory of Economic Growth. *The quarterly journal of economics*, 70(1):65–94, 1956.
3. V. Böhm and L. Kaas. Differential savings, factor shares, and endogenous growth cycles. *Journal of Economic Dynamics and Control*, 24(5-7):965–980, 2000.
4. S. Brianzoni, C. Mammana, and E. Michetti. Complex Dynamics in the Neoclassical Growth Model with Differential Savings and Non-Constant Labor Force Growth. *Studies in Nonlinear Dynamics & Econometrics*, 11(3), 2007.
5. S. Brianzoni, C. Mammana, and E. Michetti. Variable elasticity of substituition in a discrete time Solow–Swan growth model with differential saving. *Chaos, Solitons & Fractals*, 45(1):98–108, 2012.
6. S. Brianzoni, C. Mammana, and E. Michetti. Local and Global Dynamics in a Discrete Time Growth Model with Nonconcave Production Function. *Discrete Dynamics in Nature and Society*, 2012, 2012.
7. F. Grassetti, C. Mammana, and E. Michetti. On the Effect of Labour Productivity on Growth: Endogenous Fluctuations and Complex Dynamics. *Discrete Dynamics in Nature and Society*, 2018:1–9, 2018.

8. G. Kaniadakis. Non-linear kinetics underlying generalized statistics. *Physica A: Statistical Mechanics and its Applications*, 296(3):405–425, 2001.

* **e–mail:** *mauromaria.baldi@unimc.it*

Difference equations related to Thue-Morse, Fibonnaci and Rudin-Shapiro sequences

Francisco Balibrea*

Thue-Morse, Fibonacci and Rudin-Shapiro sequences appear in many combinatorial, symbolic and physical problems. They are interesting since such problems can be dealt and understood by non-linear difference equations. Problems of transmission of waves introduced by Avishai and Berend in several papers published in Physical Review, can be treated by the plane system of equations

$$T_{a,b}(x,y) = (x(a-x-y), bxy)$$

where a, b are real parameters.

In the case of Fibonacci sequences again in the transmission of waves can be understood by the system of equations in $\mathbb{R}^3$

$$F_{a,b,c}(x,y,z) = (ay, bz, cy(z-x))$$

where a, b, c are real parameters.

Rudin-Shapiro sequences produce in the solution of a one dimensional Schrodinger similar systems.

The main aim of the talk is to study some properties of the solutions of the formers systems under the dynamics point of view.

References

1. Y. Avishai, D.Berend, Transmission though Fibonacci chain. *Phys. Rev.B.* **45** (1991), 6873-6880.

2. F.Balibrea, J.L.García-Guirao, J.L.Lampart, J.Llibre, Dynamics of a Lotyka-Volterra map. *Fundam. Math.* 191, (2006),265-279.

* **e–mail:** *balibrea@um.es*

Dynamics of Population Growth: Stability Analysis of the Ricker Model

Dorsaf Bazzaa*, Jose S. Cánovas

In this article, we investigate the Ricker equation, a mathematical model that describes population dynamics through the function $f(x, r, \alpha) = x^{\alpha} e^{r(1-x^{\gamma})}$. Here, x represents the population size, while parameters r, α, and γ influence growth rates and ecological interactions. We analyze the fixed points of the model and their stability as the parameter α varies. Our findings reveal distinct cases: when $\alpha = r\gamma + 1$, the system exhibits two fixed points, $x = 0$ and $x_1 = 1$; when $\alpha > r\gamma + 1$, three fixed points emerge, including $x_0 = \left(\frac{\alpha-1}{r\gamma}\right)^{\frac{1}{\gamma}}$. The stability of these points is contingent on the relationship between x_0 and 1. Additionally, we establish that the Schwarzian derivative of the function is negative for all $x > 0$, indicating the presence of a unique stable fixed point under certain conditions. This work contributes to the understanding of population dynamics and the implications of parameter variations in ecological models.

References

1. H. El-Metwally, I. M. Alsulami, and M. Y. Hamada, ”On generalized discrete Ricker map,” *AIMS Mathematics*, *AIMS Mathematics*, (October 2024),

2. L. Block, J. Keesling, S. H. Li and K. Peterson, ”An improved algorithm for computing topological entropy”, *J. Stat. Phys* **55**, (1989).

3. L.S. Block and W.A. Coppel, ”Dynamics in one dimension, Lecture Note in Mathematics 1513,” *Springer-Verlag*, (Berlin, 1992).

4. Yu. A. Kuznetsov, Elements of applied bifurcation theory. Third edition, Applied Mathematical Sciences **112**, Springer-Verlag, New York, 2004.

* **e–mail:** *didbazza@gmail.com*

Existence proof of a chaotic attractor for the Lozi mappings

Khadija Ben Rejeb*

Strange attractors have been observed numerically in many interesting models such as Lorenz systems, Hénon system, and other models that can be proposed for chaotic dynamics of some phenomena like turbulence, weather prediction, ... In order to obtain the simplest models that can describe chaotic phenomena, 2D piecewise-linear maps are extensively studied including the famous Lozi mapping of the plane given by the formula

$$L(x,y) = (1 + y - a|x|, bx); \tag{1}$$

where a and b are two real parameters. A *trapping region* of L is a compact subset G of the plane that is mapped into its interior. So, $(L^n(G))_{n\geq 0}$ is a decreasing sequence converging to some closed subset A. If there is no closed subset of A which is limit of a same sequence, then A is called an *attractor* for L. The *unstable manifold* at the fixed point $X = (\frac{1}{1+a-b}, \frac{b}{1+a-b})$ is defined by the set

$$W^u_X := \{B \in \mathbb{R}^2 \ : \ L^{-m}(B) \longrightarrow X, \ as \ m \longrightarrow +\infty\}. \tag{2}$$

Consider the region of parameters S defined by

$$S := \{0 < b < 1, \ b + 1 < a < 2 - \frac{b}{2}\}. \tag{3}$$

In [1], Misiurewicz proved that for some subset of S, an attractor A of L exists, such that $A = \overline{W^u_X}$, A is the closure of some orbit, and the restriction $L_{|A}$ is weakly mixing. Such attractor is called as *hyperbolic attractor* (see also [2]).

In this talk, we prove that for the whole region S, the map L has a chaotic attractor $A = \overline{W^u_X}$, and the restriction $L_{|A}$ is Li-Yorke chaotic. For this aim, we prove the following properties.

- $W^u_X = I_1 \cup \cup_{n=0}^{\infty} L^n(J_1)$, $W^s_X = I_2 \cup \cup_{n=0}^{\infty} L^{-n}(J_2)$, (see Figure 1).
- $A = \cap_{n=0}^{\infty} L^n(G) = \limsup_n L^n(G \cap P^+) = \overline{\cup_{n\in\mathbb{Z}} L^n(V_X \cap A)}$, for every neighborhood $V_X \subset int(G)$, (see Figure 2).
- $A = \overline{W^s_X \cap A}$.
- The restriction $L_{|A}$ is Li-Yorke chaotic.

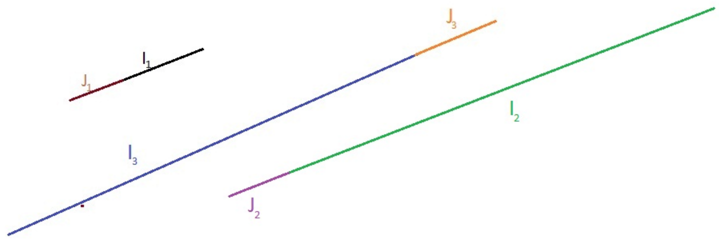


Figure 1. The unstable manifold W^u_X is equal to the segment I_1 union the union of all positive iterates $L^n(J_1)$. Whereas, the unstable manifold W^u_Y is equal to the half axis I_3 union the union of all positive iterates $L^n(J_3)$ of the segment J_3. The stable manifold W^s_X is equal to the half axis I_2 union the union of all negative iterates $L^{-n}(J_2)$ of the segment J_2

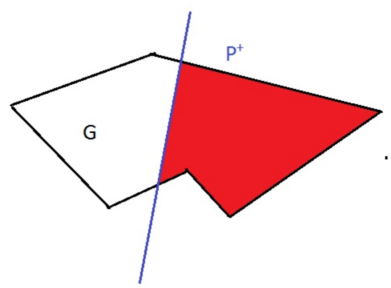


Figure 2. The trapping region G is like (homeomorphic to) the shape of this figure, and its intersection with the half plane P^+ is equal to the red region. So, the attractor A is equal to the limit superior of the n-th iterate of this red region by the map L.

References

1. Misiurewicz, M. (1980). Strange attractor for the Lozi mapping. Ann.N.Y. Acad. Sci., (357), 348-358.

2. Misiurewicz, M and Stimac. S. (2018). Lozi-like maps. Discrete Continuous Dyn Syst., 38(6), 2965-2985.

* **e–mail:** *kbrjeb@yahoo.fr*

Topological sequence entropy of tree maps

Aymen Daghar*, Jose S. Cánovas

We prove that a zero topological entropy continuous tree map always displays zero topological sequence entropy when it is restricted to its non-wandering and chain recurrent sets. In addition, we show that a similar result is not possible when the phase space is a dendrite even when we consider only the restriction on the set of periodic points.

References

1. Cánovas, J. S, Daghar, A. Topological sequence entropy and topological dynamics of tree maps. Journal of Mathematical Analysis and Applications, 525(1) (2023), 127-133.

* **e–mail:** *aymen.daghar@isgb.ucar.tn*

Attracting fixed points for the Buchner-Żebrowski equation: the role of negative Schwarzian derivative

José Ginés Espín Buendía*, Víctor Jiménez López

Roughly speaking, if $I \subset \mathbb{R}$ is an interval, we say that a C^3 map $h : I \to I$ *belongs to the class* S if it is unimodal, has a unique fixed point and its Schwarzian derivative is negative. The well-known Singer-Allwright theorem states that maps *belonging to the class* S have a locally attracting fixed point only if this attractor is global.

If, given a map h as above, the one-dimensional system $x_{n+1} = h(x_n)$ presents a repelling fixed point u, one can perturb the system to a multi-dimensional one

$$x_{n+1} = (1-\alpha)h(x_n) + \alpha x_{n-k} \tag{4}$$

where $0 < \alpha < 1$ (the so-called Buchner-Zebrowski equation) with, when k is even, (possibly) u being a local attractor.

In this context the following question seems natural: does local attraction for the perturbed system (4) imply global attraction? We present our investigation in this regard: a negative answer to the question (in general) is given. Furthermore, we shall discuss the role that the Schwarzian derivative plays in this research and why it allows us to conjecture that a positive answer to the question is plausible for large values of k.

* **e–mail:** *josegines.espin@um.es*

Discrete population models and dispersal: Part II

Daniel Franco*

Understanding the effect of dispersal on the total biomass of spatially fragmented populations is a key question in ecology. In the talk of this conference entitled *Discrete population models and dispersal: Part I* will be shown that the possible response scenarios of the overall population size to increasing dispersal are either monotonic or hump-shaped when considering two-patch discrete-time population dynamics with local dynamics described by Beverton-Holt maps. Here, we will discuss how modifying the model to include certain aspects relevant form the ecological point of view can reduce or enlarge this number of possible response scenarios. This work has been done in collaboration with C. Guiver (Napier E. University, UK), H. Logemann (U. Bath, UK), M. Marvá (U. Alcalá, Spain), J. Perán (UNED, Spain), and J. Segura (EADA Business School, Spain).

References

1. D. Franco, C. Guiver, H. Logemann, J. Pern. Dynamic properties of a class of forced positive higher-order scalar difference equations: persistency, stability and convergence. *Journal of Difference Equations and Applications*, (2025) 1-35.
2. D. Franco, J. Perán, J. Segura. New insights into the combined effect of dispersal and local dynamics in a two-patch population model. *Journal of Theoretical Biology*, 595 (2024) 111942.
3. J. Segura, M. Marvá, D. Franco. Should I link or should I not? Dispersal in multi-patch metapopulations: the impact of patch number and network topology. *Journal of Mathematical Biology* (submitted).

* **e–mail:** *dfranco@ind.uned.es*

Weird quasiperiodic attractors in two-dimensional PWL and PWS triangular maps

Laura Gardini*, Davide Radi, Noemi Schmitt, Iryna Sushko, Frank Westerhoff

The class of two-dimensional discontinuous piecewise linear (PWL) maps defined by functions with the same fixed point have a particular kind of dynamics [1,3]. There may be attracting sets consisting in segments filled with nonhyperbolic cycles or quasiperiodic trajectories, that are non generic and structurally unstable. The generic occurrence is some attractors with a strange structure that we call weird quasiperiodic attractors (WQAs).

In this talk, we show that the class of maps having dynamics characterized by the existence of WQAs includes also some discontinuous piecewise smooth (PWS) maps. This comes immediately from the fact that PWL discontinuous maps in our class are topologically conjugate to discontinuous PWS maps with a component that is a linear-fractional function. This leads to the class of discontinuous triangular maps. A few examples in this class are then used to clarify some of the properties of this kind of attractors.

References

1. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, On the emergence and properties of weird quasiperiodic attractors, submitted for publication (2025). https://arxiv.org/abs/2503.11264.
2. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, Dynamics of 1D discontinuous maps with multiple partitions and linear functions having the same fixed point. An application to financial market modeling, submitted for publication (2025). https://arxiv.org/abs/2503.20449.
3. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, Abundance of weird quasiperiodic attractors in piecewise linear discontinuous maps, submitted for publication (2025). https://arxiv.org/abs/2504.04778.

* **e–mail:** *laura.gardini@uniurb.it*

On the dynamics of a family of planar discontinuous piecewise linear maps

Armengol Gasull*

We consider the family of discontinuous piecewise linear maps

$$F_\alpha(x,y) = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} x - \operatorname{sign}(y) \\ y \end{pmatrix}.$$

For the special cases $\alpha \in \mathcal{A} := \{\pi/3, \pi/2, 2\pi/3, 4\pi/3, 3\pi/2, 5\pi/3\}$, they are pointwise periodic but not globally periodic. In particular, their sets of periods are unbounded. For each of these cases, in [1], we prove the existence of a first integral whose energy levels are discrete and, furthermore, whose level sets are bounded sets whose interior is a necklace formed by a finite number of open tiles of a certain regular or uniform tessellation. We also describe the dynamics of the maps on each invariant set of tiles in geometric terms. It depends explicitly on the value of the first integral on each of them.

The general properties of the maps F_α with $\alpha \notin \mathcal{A}$, being a rational multiple of π, are still not completely known, see [2,3]. For them we present several partial results given in [2] and also propose some open problems. All the results are obtained in collaboration with Anna Cima, Vctor Maosa and Francesc Maosas.

References

1. A. Cima, A. Gasull, V. Mañosa, F. Mañosas, Pointwise periodic maps with quantized first integrals, *Commun. Nonlinear Sci. Numer. Simulat.* **108** (2022), 106150 (26 pages)

2. A. Cima, A. Gasull, V. Mañosa, F. Mañosas, On some rational piecewise linear rotations, *J. Difference Equ. Appl.* **30** (2024), 1577–1589.

3. A. Goetz, A. Quas, Global properties of a family of piecewise isometries, *Ergod. Th. & Dynam. Sys.* **29** (2009), 545–568.

* **e–mail:** *armengol.gasull@uab.cat*

On numerical analysis of car-following models with discrete fractional operators

Ewa Girejko*, Katarzyna Topolewicz, Agnieszka B. Malinowska

The purpose of the car following model (CFM) is to control the driver's behavior with respect to the preceding vehicle in the same lane. In the basic approach, the state of a given car - that is, its position and speed - is controlled to bring its state as close as possible to that of the leading car [1,2,3]. The application of fractional operators to car-following models provides a more nuanced and realistic representation of traffic dynamics by incorporating memory effects and non-local interactions, and a better understanding of human behavior. This approach holds promise for advancing the accuracy and applicability of car-following models in simulating and understanding complex traffic scenarios. During the talk, the car-following model with fractional Gronwald-Letnikov and Caputo operators will be considered. We indicate and prove stability conditions for systems with these operators, showing that they are independent of the operator used. An exhaustive numerical analysis that verifies and illustrates theoretical investigations will be provided.

References

1. H. Greenberg, An analysis of traffic flow, *Operations research*, **4** (1), Informs, (1959), 79-85.
2. P.I. Richards, Shock waves on the highway, *Operations research*, **4** (1), Informs, (1956), 42-51.
3. T. Nagatani, T.: Traffic jams induced by fluctuation of a leading car, *Physical Review E*, **61** (4), APS (2000), 3534.

* **e–mail:** *e.girejko@pb.edu.pl*

Connecting stationary fronts of Nagumo lattice differential equation with a functional equation

Jakub Hesoun*

Motivated by a study of stationary states in a semidiscrete formulation of the classical reaction-diffusion equation with bistable dynamics (known as Nagumo lattice differential equation)

$$u_i' = d\,(u_{i-1} - 2u_i + u_{i+1}) + g(u_i), \quad i \in \mathbb{Z}, \tag{5}$$

where $d > 0$ and g is a cubic-like function satisfying $g(0) = g(a) = g(1) = 0$ for some $a \in (0, 1)$, we derive a functional equation

$$\frac{f(u) + f^{-1}(u)}{2} = \varphi(u), \tag{6}$$

with φ being function associated with reaction term g.

In this talk, we present the relation between the functional equation (6) and a mirroring scheme – a geometric interpretation of the second-order difference equation given by (5) with zero left-hand side. We establish sufficient conditions for the local solvability and uniqueness of the solution, and provide explicit global solutions over the interval $[0, 1]$ for certain functions φ. As a consequence of our study of the functional equation (6), we obtain stationary wave solutions of the original model.

This is a joint work with Petr Stehlík and Jonáš Volek.

References

1. J. Hesoun, P. Stehlík, and J. Volek. Unbounded asymmetric stationary solutions of lattice Nagumo equations, *Qualitative Theory of Dynamical Systems*, **23(2)**, (2023).

2. J. Hesoun, P. Stehlík, and J. Volek. Mirroring in lattice equations and a related functional equation, *Electronic Journal of Qualitative Theory of Differential Equations*, **65**, (2024).

* **e–mail:** *hesounj@kma.zcu.cz*

Sensitivity analysis for attractors of a stochastic piecewise ARMA(2,2)-type asset-price model

Jochen Jungeilges*, Tatyana Perevalova, Iryna Sushko, Fabio Tramontana

Based on [1] we specify a stochastic financial market model in which fundamentalists and two types of chartists, i.e. trend followers and contrarians, trade simultaneously. The stochastic asset price process takes the form of a piecewise ARMA(2,2) process. The deterministic skeleton of this process is constituted by a two-dimensional discontinuous map defined by two linear functions. One of the functions operates in the partition between two (parallel) discontinuity lines, whereas the other acts outside this partition. Despite its linearity, the deterministic skeleton exhibits complex dynamics and coexistence of various attractors. Our analysis of the stochastic system focuses on the sensitivity of fixed points and cycles to noise. To facilitate our analysis, the *controllable-canonical form* of the ARMA(2,2) model is derived. Then, the sensitivity of the attractors of the resulting minimal-dimensional Markovian model to additive noise is studied.All results are obtained by combining the stochastic sensitivity function (SSF) approach due to [2] with concepts and techniques employed by [1] in their study of the deterministic 2D discontinuous financial market model. We accentuate the economic interpretation of phenomena revealed in the course of the sensitivity analysis and expound aspects of the statistical estimability of the asset price model.

References

1. I. Sushko, F. Tramontana, Regular and chaotic dynamics in a 2D discontinuous financial market model with heterogeneous traders, *Mathematics and Computers in Simulation*, 2024, DOI: 10.1016/j.matcom.2024.01.021.

2. G.N. Mil'Shtein, L.B. Ryashko, The first approximation in the quasipotential problem of stability of non-degenerate systems with random perturbations, *Journal of Applied Mathematics and Mechanics*, 1995, 59(1), 47–56

* **e–mail:** *jochen.jungeilges@uia.no*

Wave Propagation and Global Dynamics of Invasion via Lattice Difference Equations

Eddy Kwessi*

We develop and analyze a lattice difference equation (LDE) framework to model the spatial dynamics of Wolbachia invasion in mosquito populations. This framework extends beyond classical integro-difference and reaction-diffusion models by incorporating spatial discreteness and habitat fragmentation more faithfully, making it well-suited for urban and patchy landscapes. The model integrates a biologically motivated growth function that exhibits bistability and an Allee threshold, arising from cytoplasmic incompatibility and relative fitness costs of Wolbachia infected hosts. We rigorously prove the existence of a global attractor, characterize the local stability of equilibria, and demonstrate the existence of traveling wave solutions. A key focus is on how dispersal kernels ranging from Gaussian to Cauchy interact with the Allee effect to influence wave formation, propagation speed, and invasion success. Our numerical simulations reveal that long-tailed kernels can overcome the Allee threshold through seeding effects, significantly accelerating wave fronts. These findings have direct implications for vector control strategies, informing optimal release thresholds and spatial targeting in heterogeneous environments. This work bridges theory and application, providing both analytical insights and computational tools for understanding spatial epidemiology in discrete habitats

References

1. Penelope A Hancock and et al. Modeling fitness costs and cytoplasmic incompatibility in wolbachia-infected hosts. *Journal of Theoretical Biology*, 2011.
2. Ary A Hoffmann and et al. Successful establishment of wolbachia in aedes populations to suppress dengue transmission. *Nature*, 2011.
3. P.E. Kloeden. Attractors of deterministic and random lattice difference equations. *Stochastics and Dynamics*, 22(2):2240006, 2022.
4. J. Yu and B. Zheng. Modeling wolbachia infection in mosquito population via discrete dynamical models. *Journal of Difference Equations*, 25(11):1549–1564, 2019.

* **e–mail:** *ekwessi@trinity.edu*

Non-constant periodic solutions of the Ricker model with periodic parameters

Fangfang Liao*, Na Sun, Yu Gu

In this talk, we will introduce some new results about the Ricker model

$$x_{n+1} = x_n \exp[r_n(1 - x_n)], \quad n = 0, 1, 2 \ldots, \qquad (*)$$

where $x_0 \geq 0$, and $\{r_n\}_{n=0}^{\infty}$ is a sequence of positive ω-periodic numbers. Some sufficient conditions on the existence of non-constant periodic solutions for equation $(*)$ are given. For the special case of period-two parameters, we show that $(*)$ has at most two non-constant 2-periodic solutions. We also obtain necessary and sufficient conditions for $(*)$ to have no, a unique, or exact two 2-periodic solutions, respectively. Examples are also given to illustrate our main results at last.

This is joint work with professors Xiaoping Wang, Fulai Chen.

* **e–mail:** *liaofangfang1981@126.com*

On totally periodic ω-limit sets for monotone maps on regular curves

Amira Mchaalia*

An ω-limit set of a continuous self-mapping of a compact metric space X is said to be totally periodic if all of its points are periodic.

Let X be a regular curve and let $f : X \to X$ be a monotone map. We show that any totally periodic ω-limit set is finite. This result extends that of Askri and Naghmouchi [1] established whenever f is one-to-one continuous map on regular curves (in particular for f homeomorphism).

References

1. G. Askri and I. Naghmouchi, On totally periodic ω-limit sets in regular continua, Chaos Solitons Fractals 75 (2015), 91-95.

* **e–mail:** *emira.mchaalia@gmail.com*

Logistic fractional variable order discrete-time equation with distributed delays

Dorota Mozyrska*, Malgorzata Wyrwas

This paper investigates the discrete-time logistic fractional variable-order equation with distributed delays. Motivated by the increasing relevance of fractional-order models in the description of memory and hereditary properties in complex systems [1], we extend the discrete-time fractional framework by incorporating both variable-order dynamics and infinite distributed delays, offering a more realistic approach. The study builds on recent advances in the theory of discrete fractional systems with Caputo-type operators [2,3] and addresses the critical issue of proper initialization for systems with infinite memory [4,5]. We provide a comprehensive analysis of the stability of solutions. Numerical simulations based on piecewise constant order functions and geometric memory kernels are presented to illustrate the theoretical results and to show the delicate balance between fractional dynamics and delay-induced phenomena [6].

References

1. R. Hilfer, *Applications of Fractional Calculus in Physics*, World Scientific Publishing Company, Singapore, 2006.

2. D. Mozyrska and M. Wyrwas, Stability of linear systems with Caputo fractional-, variable-order difference operator of convolution type, in *41st Int. Conf. Telecommun. Signal Process. (TSP)*, IEEE, 2018, 1–4.

3. D. Mozyrska and M. Wyrwas, Stability analysis of discrete-time fractional-order Caputo type Lorenz system, in *42nd Int. Conf. Telecommun. Signal Process. (TSP)*, IEEE, 2019, 149–153.

4. R. Garrappa and E. Kaslik, On the initialization of fractional delay differential equations, *Mathematics* **8** (2020), 1681.

5. E. Kaslik and S. Sivasundaram, Analytical and numerical methods for the stability analysis of fractional-order delay differential equations, *J. Comput. Appl. Math.* **236** (2011), 2473–2481.

6. D. Mozyrska and M. Wyrwas, Stability of linear discrete-time systems with the Caputo fractional-, variable-order h-difference operator of convolution type, in *Proc. Int. Conf. Fractional Differentiation and its Applications (ICFDA)*, 2018.

* **e–mail:** *d.mozyrska@pb.edu.pl*

Weird quasiperiodic attractors in two-dimensional PWL and PWS triangular maps

María Muñoz-Guillermo*, Jose S. Cánovas

It is usual to approach the bifurcation problem using numerical and computational methods. In this case we will address the possibility of giving sufficient analytical conditions that allow us to conclude the existence of codimension-1 bifurcations of the fold, transcritical, pitchfork and flip type for a dynamical system which is governed by the equation $x_{n+1} = f(x_{n-1}, x_n, a)$, where $a \in \mathbb{R}$ is a parameter. Thus, if these conditions are met, we will be able to affirm the existence of the corresponding bifurcation.

References

1. K. T. Alligood, T. D. Sauer and J. A. Yorke, *Chaos: An Introduction to Dynamical Systems.* Springer-Verlag, New York, (1997).

2. V. I. Arnolď, *Dynamical Systems V. Bifurcation Theory and Catastrophe Theory*, volume 5 of *Encyclopaedia of Mathematical Sciences* Springer Berlin, Heidelberg. (1994). https://doi.org/10.1007/978-3-642-57884-7

3. F. Balibrea, H. M. Oliveira, and J. C. Valverde. *Topological equivalences for one-parameter bifurcations of scalar maps.* Journal of Nonlinear Science, **27** (2), 661-685 (2017).

4. J. Carr, *Applications of Centre Manifold Theory*, volume 35. Springer New York, NY (1982). https://doi.org/10.1007/978-1-4612-5929-9

5. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, volume 42 Springer New York, NY, 1983.

6. A. Kelley, *Stability of the center-stable manifold* Journal of Mathematical Analysis and Applications, **18**, Issue 2, (1967), 336-344.

7. Y. A. Kuznetsov, *Elements of Applied Bifurcation Theory.* Springer . Applied Mathematical Series, **112** (2004).

8. H.W. Lorenz, *Nonlinear Dynamical Economics and Chaotic Motion*, Springer (1993).

9. D. Whitley, *Discrete Dynamical Systems in Dimensions One and Two*, Bulletin of the London Mathematical Society, **15** (1983) 177-217.

10. S. Wiggins, *Local Bifurcations. In: Introduction to Applied Nonlinear Dynamical Systems and Chaos. Texts in Applied Mathematics*, vol 2. Springer, New York, NY. (1990) https://doi.org/10.1007/978-1-4757-4067-7_4.

* **e–mail:** *maria.mg@upct.es*

Codimension-one and -two bifurcations and stability of certain second order rational difference equation with arbitrary parameters

Naida Mujić* and Zenan Šabanac

Motivated by previous investigations that analyzed the boundedness of positive solutions, global stability, and the occurrence of Neimark-Sacker bifurcation in specific parameter cases, we comprehensively investigate the dynamics of the second-order rational difference equation

$$x_{n+1} = C + A\frac{x_n^k}{x_{n-1}^p},$$

where parameters $k,\ p,\ A,\ C$ and initial conditions $x_{-1},\ x_0$ are positive real numbers. We provide a complete topological classification of fixed (equilibrium) points and examine the local behavior of orbits in the neighborhood of these points, which, to our knowledge, has not been previously studied in the entire admissible parameter space. Our research has discovered highly complex and rich dynamic behavior, ranging from the occurrence of supercritical and subcritical Neimark-Sacker bifurcations in different parameter spaces to the appearance of codimension-2 bifurcations in the case of 1-1 strong resonance. A very interesting situation appears when one of the equilibria is nonhyperbolic in specific parameter space; direct calculations have shown that both the first and second Lyapunov coefficients are equal to zero, implying that this equilibrium is a Hopf point of codimension at least 3. This strongly suggests the complex behavior of the studied equation, which numerical simulations have also confirmed.

References

1. M. Berkal, J. F. Navarro, Qualitative study of a second order difference equation, *Turkish J. Math.* **47** (2023), no. 2, 516–527.

2. E. Beso, S. Kalabusic, N. Mujic, E. Pilav, Boundedness of solutions and stability of certain second-order difference equation with quadratic term, *Adv. Difference Equ.* (2020), Paper No. 19, 22 pp.

3. S. Elaydi, R. Luis, M. Guzowska, Bifurcation and Invariant Manifolds of the Logistic Competition Model, *J. Difference Equ. Appl.* **00** (2010), no. 00, 1–21.

4. T. Khyat, M. R. S. Kulenovic, E. Pilav, The Neimark-Sacker bifurcation and asymptotic approximation of the invariant curve of a certain difference equation, *J. Comput. Anal. Appl.* **23** (2017), no. 8, 1335–1346.

5. M. R. S. Kulenovic, G. Ladas, *Dynamics of Second Order Rational Difference Equations, with Open Problems and Conjectures*, Chapman & Hall/CRC Press, 2001.

6. Yu. A. Kuznetsov, *Elements of Applied Bifurcation Theory*, 4th ed., Springer Cham, 2023.

7. Yu. A. Kuznetsov, H. G. E. Meijer, Numerical normal forms for codim 2 bifurcations of fixed points with at most two critical eigenvalues, *SIAM J. Sci. Comput.* **26** (2005), no. 6, 1932–1954.

8. Yu. A. Kuznetsov, H. G. E. Meijer, Numerical bifurcation analysis of maps: From theory to software, *Cambridge Monogr. Appl. Comput. Math.*, 34 Cambridge University Press, Cambridge, 2019.

9. K. Murakami, The Invariant curve caused by Neimark-Sacker bifurcation, *Dynamics of Continuous, Discrete and Impulsive Systems*, **9** (2002), no. 1, 121–132.

10. J. Sotomayor, L. F. Mello, D.C. Braga, Lyapunov Coefficients for degenerate Hopf bifurcations, (2007). https://doi.org/10.48550/arXiv.0709.3949.

11. S. Stevic, On the recursive sequence $x_{n+1} = \alpha + \frac{x_n^p}{x_{n-1}^r}$, *Discrete Dyn. Nat. Soc.* (2007), Art. ID 40963, 9 pp.

12. G. Tigan, O. Brandibur, E. A. Kokovics, L. F. Vesa, Degenerate Chenciner bifurcation revisited, *Internat. J. Bifur. Chaos Appl. Sci. Engrg.* **31** (2021), no. 10, Paper No. 2150160, 10 pp. https://doi.org/10.1142/S0218127421501601.

13. G. Tigan, S. Lugojan, L. Ciurdariu, Analysis of degenerate Chenciner bifurcation, *Internat. J. Bifur. Chaos Appl. Sci. Engrg.* **30** (2020), no. 16, Paper No. 2050245, 11 pp. https://doi.org/10.1142/S0218127420502454.

14. K. Yagasaki, Melnikov's method and codimension-two bifurcations in forced oscillations, *J. Differ. Equ.* **185** (2002), 1–24.

* **e–mail:** *nmujic@etf.unsa.ba*

On the dynamics of a family of max-type difference equations

Daniel Nieves Roldán*, Antonio Linero Bas

In the last decades, there has been a special interest in studying the behaviour of models described by difference equations with maximum. One of the most famous families of difference equations with maximum is the so-called *generalized Lyness max-type equations*, which are given by

$$x_{n+1} = \frac{\max\{x_n^k, A\}}{x_n^l x_{n-1}^m}, \quad n = 0, 1, \ldots, \tag{7}$$

where $k, l, m \in \mathbb{Z}$ and $A, x_{-1}, x_0 \in (0, \infty)$.

The main goal of this talk is to advance in the knowledge of the dynamics of the family of max-type difference equations

$$x_{n+1} = \frac{\max\{x_n, A\}}{x_n x_{n-1}}, \tag{8}$$

where $A, x_{-1}, x_0 \in (0, \infty)$.

To do so, we study in detail concrete piecewise linear difference equations which are topologically conjugate to Equation (8). Indeed, we describe completely the dynamics of the family and add some new features in terms of complete integrability. Moreover, we tackle some other cases of the generalized Lyness max-type equations which exhibits interesting dynamics.

References

1. A. Cima, A. Gasull, V. Mañosa, Global periodicity and complete integrability of discrete dynamical systems, *J. Difference Equ. Appl.* **12**(2006), 697-726.

2. J. Feuer, E.J. Janowski, G. Ladas, and C. Teixera, Global behavior of solutions of $x_{n+1} = \frac{\max\{x_n, A\}}{x_n x_{n-1}}$, *J. Comput. Anal. Appl.* **2**(2000), 237-252.

3. J. Feuer, Periodic solutions of the Lyness max equation, *J. Math. Anal. Appl.* **288**(2003), 147-160. Doi: 10.1016/S0022-247x(03)00587-0

4. A. Gelisken, C. Cinar, I. Yalcinkaya, On the periodicity of a difference equation with maximum, *Discrete Dyn. Nat. Soc.* (2008), Article ID 820629, 11 pages. Doi: 10.1155/2008/820629

5. A. Linero Bas, D. Nieves Roldan, On the relationship between Lozi maps and max-type difference equations, *J. Difference Equ. Appl.* **29**(2022), no. 9-12, 1015-1044. Doi: 10.1080/10236198.2022.2120393

* **e–mail:** *daniel.nieves@um.es*

Noise-induced de-synchronization in dynamic stochastic consumption model

Tatyana Perevalova*, Jochen Jungeilges

We study the stochastic version of the consumption model proposed in [1] in which interacting agents form preferences endogenously and take consumption decisions in discrete time:

$$x_{1t+1} = \frac{b_1}{p_x p_y}(a_1 x_{1t}(b_1 - p_x x_{1t}) + D_{12} x_{2t}(b_2 - p_x x_{2t})) + \varepsilon \xi_{1,t}, \tag{9}$$

$$x_{2t+1} = \frac{b_2}{p_x p_y}(a_2 x_{2t}(b_2 - p_x x_{2t}) + D_{21} x_{1t}(b_1 - p_x x_{1t})) + \varepsilon \xi_{2,t}, \tag{10}$$

where x_{1t} and x_{2t} denote units of the commodity x consumed by the first and second individual at time t, p_x and p_y are the unit prices of the commodities x and y, while the parameters b_1 and b_2 represent the nominal income of the first and second individual. Prices and incomes are assumed to be constant in time. The real, strictly positive, parameters α_1, α_2 and D_{12}, D_{21} are referred to as *learning* and *influence* parameters respectively. The shocks $\xi_{1,t}$ and $\xi_{2,t}$ are assumed to be realizations of independently and identically distributed standard Gaussian random variables occurring at time t. The scalar constant $\varepsilon \geq 0$ functions as a noise intensity.

In a first step, we carry out an analysis of attractors and bifurcations when the value of one of the bifurcation parameters depends on the second one as follows: $D_{21} = 4(D_{12} - \alpha_2) + \alpha_1$. As was shown in [2], in this case the synchronization of variables occurs on one of the attractors, so that $x_2 = 2x_1$.

On the second stage of the analysis, we focus on the description of noise-induced de-synchronization and transitions between attractors. For a constructive description of stochastic phenomena, the stochastic sensitivity function technique [3,4] and the associated confidence domain method are used.

References

1. W. Gaertner and J. Jungeilges A non-linear model of interdependent consumer behaviour. *Economics Letters* **27** (1988), 145-150.

2. E. Ekaterinchuk, J. Jungeilges, T. Ryazanova and I. Sushko Dynamics of a minimal consumer network with bi-directional influence. *Communications in Nonlinear Science and Numerical Simulation* **58** (2018), 107-118

3. I. Bashkirtseva and L. Ryashko Stochastic sensitivity of regular and multi-band chaotic attractors in discrete systems with parametric noise. *Physics Letters, Section A: General, Atomic and Solid State Physics* **381(37)** (2017) 3203-3210.

4. I. Bashkirtseva, L. Ryashko and I. Tsvetkov Sensitivity analysis of stochastic equilibria and cycles for the discrete dynamic systems. *Dynamics of Continuous, Discrete and Impulsive Systems Series A: Mathematical Analysis* **17** (2010), 501-515.

* **e–mail:** *tatiana.perevalova@ec.unipi.it*

Multifractal dimensions of invariant subsets for piecewise monotonic maps

Peter Raith*

Let $T : [0,1] \to [0,1]$ be an expanding piecewise monotonic map. This means that there exists a finite family $\mathcal{Z}$ of pairwise disjoint open intervals with $\bigcup_{Z\in\mathcal{Z}} \overline{Z} = [0,1]$ such that $T\big|_Z$ is continuous and strictly monotonic for all $Z \in \mathcal{Z}$. Moreover, for every $Z \in \mathcal{Z}$ the map $T\big|_Z$ is differentiable and its derivative can be extended to a continuous function on the closure of Z, and we have $\inf |T'| > 1$. For a finite union of open intervals U set $A(U) := [0,1] \setminus \bigcup_{j=0}^{\infty} T^{-j}U$. Obviously this is the set of all points whose orbit never enters U. In this talk the size of $A(U)$ will be investigated.

Considering two examples motivates the definition of multifractal Hausdorff dimension. After the definition of multifractal Hausdorff dimension some results obtained together with F. Hofbauer and T. Steinberger are presented.

* **e–mail:** *Peter.Raith@univie.ac.at*

The dynamic of q-deformed logistic maps

Houssem Eddine Rezgui*, Jose S. Cánovas

In this talk, we study a two-parameter family of q-deformed logistic maps defined by $\Phi_{q,r} = \phi_q \circ f_r$, where $\phi_q(x) = \frac{1-q^x}{1-q}$ is a q-deformation on the interval $I = [0,1]$, and $f_r(x) = rx(1-x)$ is the classical logistic map with $r \in (0,4]$.

In this presentation, we focus on characterizing the parameter region where the dynamics of $\Phi_{q,r}$ is chaotic. To this end, we compute the topological entropy with fixed accuracy and estimate Lyapunov exponents over a broad range of parameters. Our analysis reveals that chaotic behavior also emerges for $q > 1$, extending previous results obtained for $q \in (0,1)$.

Additionally, we highlight how these models exhibit Parrondos paradox, where the composition of two dynamically simple maps produces a chaotic dynamical system. We will show that the parameter region q ¡ 1 is highly suitable to produce the paradox.

We conclude by presenting a generalization of the model through the study of periodic compositions involving multiple q-deformations, and we compare our results with those obtained using alternative q-deformations in the literature.

References

1. J. S. Canovas, On q-deformed logistic maps. Discrete. continuous Dyn sys-B., 27(5) (2022), 2833.
2. J. Canovas, M. Muñoz, On the dynamics of the q-deformed logistic map. Phys. Lett. A, 383 (2019), 17421754.
3. J. S. Canovas, H. E. Rezgui, Revisiting the dynamic of q-deformed logistic maps. Chaos, Solitons and Fractals, 167 (2023), 113040.
4. D. Gupta, V. Chandramouli, An improved q-deformed logistic map and its implications. Pramana J Phys (2021), 95:175.

* **e–mail:** *rezguihoussemeddinne@gmail.com*

Structure of zero entropy patterns of trees

David Rojas*, David Juher, Francesc Mañosas

The notion of *pattern* plays a central role in the theory of topological and combinatorial dynamics. Consider a family $\mathcal{X}$ of topological spaces (for instance the family of either all closed intervals of the real line, or all trees, or all graphs, or compact surfaces, etc) and the family $\mathcal{F}_{\mathcal{X}}$ of all maps $\{f : X \to X : X \in \mathcal{X}\}$ satisfying a given restriction (continuous maps, homeomorphisms, etc). Given a map $f : X \to X$ in $\mathcal{F}_{\mathcal{X}}$ which is known to have a finite invariant set P, the *pattern of* P *in* $\mathcal{F}_{\mathcal{X}}$ is the equivalence class $\mathcal{P}$ of all maps $g : Y \to Y$ in $\mathcal{F}_{\mathcal{X}}$ having an invariant set $Q \subset Y$ that, at a combinatorial level, behaves like P. That is, the relative positions of P inside X, and the way these positions are permuted under the action of g coincides with the way f acts on the points of P. In this case, it is said that every map g in the class *exhibits* the pattern $\mathcal{P}$. In in particular P is a periodic orbit of f, the pattern $\mathcal{P}$ is said to be *periodic*.

In this talk we deal with periodic patterns of continuous maps defined on trees (simply connected graphs). In particular, we present a topological characterization of patterns whose entropy is zero and present some combinatorial properties that gives a second purely combinatorial characterization of those patterns.

References

1. Ll. Alsedá, D. Juher and F. Mañosas. Topological and algebraic reducibility for patterns on trees. *Ergod. Th. & Dynam. Sys.* **35** (2015), 34–63.

2. D. Juher, F. Mañosas and D. Rojas. Characterization of the tree cycles with minimum positive entropy for any period. *Ergod. Th. & Dynam. Sys.* Published online 2025:1–44. doi:10.1017/etds.2025.11

* **e–mail:** *david.rojas@uab.cat*

Discrete population models and dispersal: Part I

Juan Segura*

Understanding the effect of dispersal on the total biomass of spatially fragmented populations is a key question in ecology. In discrete time, the question remained virtually open until the work [2] for two-patch metapopulations. However, only a description for low dispersal rates was obtained. By contrast, in continuous time, the latest research provided a complete theoretical description for all possible dispersal rates in the case of local logistic growth [2,3]. These and all previous numerical and theoretical findings determined that the possible response scenarios of the overall population size to increasing dispersal are either monotonic or hump-shaped.

Regarding the discrete-time question, we will present two important results. First, a complete description of the response scenarios to dispersal of the total biomass for local dynamics described by Beverton-Holt maps, which are the analog of local logistic growth in continuous time. Second, we will explore how dispersal asymmetry induced by limiting migration in one direction can allow managers to lead the population to any desired response scenario among the possible. This work has been done in collaboration with D. Franco (UNED, Spain), F. Hilker (U. Osnabrück, Germany), C. Grumbach (U. Osnabrück, Germany), and F. Reurik (U. Osnabrück, Germany).

The results presented in this talk will be extended to other situations relevant from an ecological perspective in the talk of this conference entitled *Discrete population models and dispersal: Part II.*

References

1. R. Arditi, C. Lobry, and T. Sari, Is dispersal always beneficial to carrying capacity? New insights from the multi-patch logistic equation, *Theor. Popul. Biol.*, **106** (2015) 45-59.

2. D. Franco and A. Ruiz-Herrera, To connect or not to connect isolated patches, *J. Theor. Biol.* **370** (2015), 72-80.

3. D. Gao and Y. Lou, Total biomass of a single population in two-patch environments. *Theor. Popul. Biol.* **146** (2022) 1-14.

* **e–mail:** *jsegura@eada.edu*

On the emergence and properties of weird quasiperiodic attractors

Gabriel Soler López*, Antonio Linero Bas

We present recent developments on minimal non-uniquely ergodic IETs with flips, including new constructions and theoretical bounds.

The existence of minimal non-uniquely ergodic oriented IETs of n intervals requires that $n \geq 4$, as shown in [7,Th. 2.12]. Moreover, for $n = 4$, there are known examples of minimal non-uniquely ergodic 4-IETs with two independent invariant measures; see [3]. Additionally, there exists a bound on the number N of independent invariant measures that an n-IET can admit. In particular, Veech proved in [7,Th. 2.12] that $N \leq \frac{n}{2}$. In fact, this bound can be refined: he showed that $N \leq \frac{R}{2}$, where R is the rank of the $n \times n$ translation matrix associated with T, which is not necessarily equal to n. Furthermore, in [8, S. IV], the reader can find an analysis showing that N is bounded by g, the genus of the suspension surface associated with T, where the genus satisfies $g = 1 + \frac{n-m}{2}$, with m being the number of so-called marked points on the surface; see [1, S. 5-6] for more details.

It is therefore interesting to seek analogous bounds for the flipped case. However, the theory of IETs with flips is not as developed as in the oriented case. The present authors have recently worked on the construction of minimal non-uniquely ergodic IETs with flips, since no such examples were previously known. Finding them had been proposed as an open problem in [2, Remark 1].

In this communication, we present recent advances in this area. First, we constructed examples of $(10, k)$-IETs for $1 \leq k \leq 10$ in [4]. This construction was later refined in [5], where we provided examples of $(6, k)$-IETs and introduced technical results that could eventually lead to constructions for (n, k)-IETs with $n \geq 5$.

Finally, we announce a key result: it is not possible to construct minimal non-uniquely ergodic (n, k)-IETs for any $n \leq 4$, marking a significant difference with the oriented case, see [6].

References

1. J. G. Espín Buendía, D. Peralta-Salas, and G. Soler López. Existence of minimal flows on nonorientable surfaces. *Discrete Contin. Dyn. Syst.*, 37(8):4191–4211, 2017.

2. C. Gutierrez, S. Lloyd, V. Medvedev, B. Pires, and E. Zhuzhoma. Transitive circle exchange transformations with flips. *Discrete Contin. Dyn. Syst.*, **26**(1):251–263, 2010.

3. M. Keane. Non-ergodic interval exchange transformations. *Israel J. Math.*, **26**:188–196, 1977.

4. A. Linero Bas and G. Soler López. Minimal non uniquely ergodic IETs with flips. *J. Differential Equations*, **360**:232–259, 2023.

5. A. Linero Bas and G. Soler López. On non uniquely ergodic minimal IETs. *Submitted*, 2024.

6. A. Linero Bas and G. Soler López. On the non existence of minimal non-uniquely ergodic 4-IETs with flips. *Submitted*, 2024.

7. W. A. Veech. Interval exchange transformations. *J. Anal. Math.*, **33**:222–272, 1978.

8. J. C. Yoccoz. Echanges d'intervalles. cours collège de france. *Https://www.college-de-france.fr/media/jean-christophe-yoccoz/UPL8726 yoccoz05.pdf*, janvier-mars 2005.

* **e–mail:** *gabriel.soler@upct.es*

On the emergence and properties of weird quasiperiodic attractors

Iryna Sushko*, Laura Gardini, Davide Radi, Noemi Schmitt, Frank Westerhoff

We recently described a specific type of attractors of two-dimensional discontinuous piecewise linear maps, characterized by two discontinuity lines dividing the phase plane into three partitions, related to economic applications [1]. To our knowledge, this type of attractor, which we call a weird quasiperiodic attractor, has not yet been studied in detail. They have a rather complex geometric structure and other interesting properties that are worth understanding better. To this end, in [2] we consider a simpler map that can also possess weird quasiperiodic attractors, namely, a 2D discontinuous piecewise linear map F with a single discontinuity line dividing the phase plane into two partitions, where two different homogeneous linear maps are defined. Map F depends on four parameters – the traces and determinants of the two Jacobian matrices. In the parameter space of map F, we obtain specific regions associated with the existence of weird quasiperiodic attractors; describe some characteristic properties of these attractors; and explain one of the possible mechanisms of their appearance.

References

1. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, On the limits of informationally efficient stock markets: New insights from a chartist-fundamentalist model. https://arxiv.org/abs/2410.21198.

2. L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, On the emergence and properties of weird quasiperiodic attractors. https://arxiv.org/abs/2503.11264.

* **e–mail:** *sushko.im@gmail.com*

List of Participants

1. Azmy S. Ackleh, University of Louisiana, USA.
   email: azmy.ackleh@louisiana.edu
2. Lluís Alsedá, Universitat Autónoma de Barcelona, Spain.
   email: lluis.alseda@uab.cat
3. Mauro Maria Baldi, University of Macerata, Italy.
   email: mauromaria.baldi@unimc.it
4. Paco Balibrea, Murcia University, Spain.
   email: balibrea@um.es
5. Dorsaf Bazzaa, Carthage University, Tunisia.
   email: didbazza@gmail.com
6. Khadija Ben Rejeb, University of Sousse, Tunisia.
   email: kbrjeb@yahoo.fr
7. Emin Bešo, University of Sarajevo, Bosnia and Herzegovina.
   email: eminbeso@pmf.unsa.ba
8. Jose S. Cánovas, Technical University of Cartagena, Spain.
   email: jose.canovas@upct.es
9. Anna Cima, Universitat Autónoma de Barcelona, Spain.
   email: anna.cima@uab.cat
10. Aymen Daghar, Carthage University, Tunisia.
    email: aymendaghar@gmail.com
11. Saber Elaydi, Trinity University, USA.
    email: selaydi@trinity.edu
12. José Ginés Espín, Murcia University, Spain.
    email: josegines.espin@um.es
13. Daniel Franco, Universidad Nacional de Educación a Distancia, Spain.
    email: dfranco@ind.uned.es
14. Laura Gardini, University of Urbino Carlo Bo, Italy.
    email: laura.gardini@uniurb.it
15. Ewa Girejko, Bialystok University of Technology, Poland.
    email: e.girejko@pb.edu.pl
16. Armengol Gasull, Universitat Autónoma de Barcelona, Spain.
    email: armengol.gasull@uab.cat
17. Gu Yu, School of Mathematics and Information Science, Xiangnan University, China.
    email: guyu@xnu.edu.cn

18. Jakub Hesoun, University of West Bohemia in Pilsen, Czech Republic.
    email: hesounj@kma.zcu.cz

19. Víctor Jiménez López, Murcia University, Spain.
    email: vjimenez@um.es

20. David Juher, Universitat de Girona, Spain.
    email: david.juher@udg.edu

21. Joachim Jungeilges, Agder University, Norway.
    email: jochen.jungeilges@uia.no

22. Senada Kalabušic, University of Sarajevo, Bosnia and Herzegovina.
    email: senadak@pmf.unsa.ba

23. Eddy Kwessi, Trinity University, USA.
    email: ekwessi@trinity.edu

24. Liao Fangfang, School of Mathematics and Information Science, Xiangnan University, China.
    email: liaofangfang1981@126.com

25. Antonio Linero Bas, Murcia University, Spain.
    email: lineroba@um.es

26. René Lozi, Université Côte d'Azur, France.
    email: Rene.LOZI@univ-cotedazur.fr

27. Víctor Mañosa, Universitat Politcnica de Catalunya, Spain.
    email: victor.manosa@upc.edu

28. Francesc Mañosas, Universitat Autónoma de Barcelona, Spain.
    email: Francesc.Manosas@uab.cat

29. Amira Mchaalia, University of Sousse, Tunisia.
    email: emira.mchaalia@gmail.com

30. Dorota Mozyrska, Bialystok University of Technology, Poland.
    email: d.mozyrska@pb.edu.pl

31. Azioune Mourad, Abdelhafid Boussouf University, Algeria.
    email: m.azioune@centre-univ-mila.dz

32. Naida Mujić, University of Sarajevo, Bosnia and Herzegovina.
    email: nmujic@etf.unsa.ba

33. María Muñoz Guillermo, Technical University of Cartagena, Spain.
    email: maria.mg@upct.es

34. Daniel Nieves Roldán, Murcia University, Spain.
    email: daniel.nieves@um.es

35. Tatiana Perevalova, University of Pisa, Italy.
    email: tatyana.perevalova@gmail.com

36. Davide Radi, Università Cattolica del Sacro Cuore, Italy.
    email: davide.radi@unicatt.it

37. Peter Raith, University of Vienna, Austria.
    email: peter.raith@univie.ac.at

38. Houssem Eddine Rezgui, Carthage University, Tunisia.
    email: rezguihoussemeddinne@gmail.com

39. David Rojas, Universitat Autónoma de Barcelona, Spain.
    email: David.Rojas@uab.cat

40. María del Carmen Ruiz Abellón, Technical University of Cartagena, Spain.
    email: maricarmen.ruiz@upct.es

41. Juan Segura, EADA Business School Barcelona, Spain.
    email: jsegura@eada.edu

42. Gabriel Soler López, Technical University of Cartagena, Spain.
    email: gabriel.soler@upct.es

43. Nadia Souayeh, University of Monastir, Tunisia.
    email: nadia.souayeh@fst.utm.tn

44. Sun Na, School of Mathematics and Information Science, Xiangnan University, China.
    email: s1067562423@163.com

45. Iryna Sushko, Institute of Mathematics, NAS of Ukraine, and Catholic University of Milan, Italy.
    email: sushko.im@gmail.com