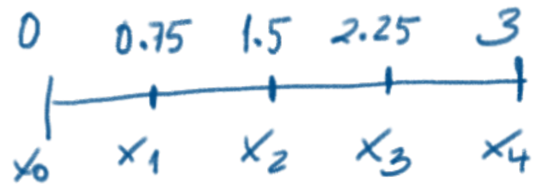
1.a

Intervalo $\Rightarrow I = [0,3]$ $\Rightarrow h = \frac{3-0}{4} = 3/4 = 0.75$

Nº de subintervalos $\Rightarrow n = 4$



$x \Rightarrow \{0, 0.75, 1.5, 2.25, 3\}$

$f(x) \Rightarrow \{0, 0.1781, 2.2331, 2.3847, 0.0253\}$

<u>TRAPECIO</u>: $\frac{h}{2} \cdot \{ f(x_0) + 2(f(x_1) + f(x_2) + f(x_3)) + f(x_4) \}$

$= \frac{3}{8} \cdot \{ 0 + 2(0.1781 + 2.2331 + 2.3847) + 0.0253 \}$

$= \frac{3}{8} \cdot 9{,}6172 = 3.6064$

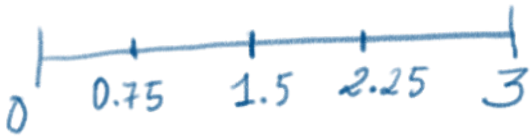
<u>SIMPSON</u>: $\frac{h}{3} \cdot \{ f(x_0) + 4 \cdot (f(x_1) + f(x_3)) + 2 f(x_2) + f(x_4) \}$

$= \frac{1}{4} \cdot \{ 0 + 4 \cdot (0.1781 + 2.3847) + 2 \cdot 2.2331 + 0.0253 \}$

$= \frac{1}{4} \; 14.743 = 3.6857$

Apartado b

b)



Para garantizar la exactitud hasta polinomios de grado 5, el número de nodos y pesos que debemos utilizar es $(n+1)$ tal que

$$2n+1 \geq 5 \Rightarrow \boxed{n=2}$$

Por tanto, el número de nodos y pesos es 3.

Usaremos la tercera de las tablas

$$w_0 = 5/9 \quad \xi_0 = -0.7746$$
$$w_1 = 8/9 \quad \xi_1 = 0$$
$$w_2 = 5/9 \quad \xi_2 = 0.7746$$

Hay que transformar el intervalo $[0,3]$ en el intervalo $[-1,1]$, usando el cambio

$$x = \frac{b-a}{2}\xi + \frac{a+b}{2} \Rightarrow dx = \frac{b-a}{2}d\xi,$$

Es decir

$$x = \frac{3}{2}\xi + \frac{3}{2} \quad y \quad dx = \frac{3}{2}d\xi$$

La integral sera

$$\int_0^3 f(x)dx = \frac{3}{2}\int_{-1}^{1} f\left(\frac{3}{2}\xi + \frac{3}{2}\right) d\xi = 3/2\int_{-1}^{1} g(\xi)\, d\xi$$

$$= \frac{3}{2}\left[\ 5/9\, g(-0.7746) + 8/9\, g(0) + 5/9\, g(0.7746)\right]$$

$$= \frac{3}{2}\left[\frac{5}{9} f\left(-\frac{3}{2}\, 0.7746 + 3/2\right) + \frac{8}{9} f(3/2) + 5/9\, f\left(3/2\, 0.7746 + 3/2\right)\right]$$

$$= \frac{3}{2}\left[\frac{5}{9} f(0.3381) + 8/9\, f(1.5) + 5/9\, f(2.6619)\right]$$

$$= \frac{3}{2}\cdot\left[\frac{5}{9}\cdot(4{,}1717\times10^{-3}) + \frac{8}{9}(2{,}231) + \frac{5}{9}(0{,}6965)\right]$$

$$= \frac{3}{2}\cdot\left[\ 2{,}3176\times10^{-3} + 0{,}3869 + 1{,}9850\right]$$

$$= \frac{3}{2}(2{,}3743) = 3{,}5614$$

Ans*3

Si usamos los nodos en cada subintervalo, deberíamos calcular, teniendo en cuenta que $\cancel{x}_k = k \cdot 3/4$

$$\frac{x_{k+1} - x_k}{2} = \frac{3}{8} \qquad \frac{x_{k+1} + x_k}{2} = \frac{(k+1)\cdot 3/4 + (k)\cdot 3/4}{2} = \frac{3}{8}\cdot(2k+1)$$

$$\int_{x_k}^{x_{k+1}} f(x)dx = \frac{3}{8}\int_{-1}^{1} f\left(3/8\,\xi + 3/8(2k+1)\right)d\xi =$$

$$3/8 \;\; \frac{5}{9}\left[f\left(-\frac{3}{8}(0.7746) + \frac{3}{8}(2k+1)\right) + 8/9 \cdot f\left(3/8\,(2k+1)\right) + 5/9\, f\left(\frac{3}{8}(0.7746) + 3/8\,(2k+1)\right)\right]$$

$$= \frac{3}{72}\left[5\cdot f\left(-0.2905 + \frac{3}{8}(2k+1)\right) + 8\cdot f\left(3/8(2k+1)\right) + 5\cdot f\left(0.2905 + \frac{3}{8}(2k+1)\right)\right]$$

$[0, 3/4] \Rightarrow 0,024021$

$[3/4, 1.5] \Rightarrow 0,7714$

$[1.5, 2.25] \Rightarrow 2,0959$

$[2.25, 3] \Rightarrow 0,7245$

$$\Rightarrow \sum_{k=0}^{3}\int_{x_k}^{x_{k+1}} f(x)dx = 3,6159.$$

2) Las reglas de cuadratura para nodos $x_0, ..., x_n$ y
y pesos $w_0, ..., w_n$ son exactas para polinomios de
hasta grado $(2n+1)$, por tanto, si queremos

Integrar $\int_1^7 x^7 dx$ de forma exacta, debe ocurrir

$$(2n+1) \geq 7$$

Por tanto, $\boxed{n=3}$; puesto que $2 \cdot 3 + 1 = 7 \geq 7$

Necesitaremos $\{x_0, x_1, x_2, x_3\}$ nodos y

$\{w_0, w_1, w_2, w_3\}$ pesos, teniendo

que usar los datos de la ultima tabla.

Hacemos el cambio $x = \frac{7-1}{2}\xi + \frac{7+1}{2} = 3\xi + 4$ y $dx = 3d\xi$,

$$\int_1^7 f(x)dx = 3\int_{-1}^1 f(3\xi+4)\,d\xi = 3\int_{-1}^1 g(\xi)\,d\xi =$$

$$3\left[0{,}3478 \cdot g(-0.8611) + 0.6521 \cdot g(-0.3399) + 0.6521 \cdot g(0.3399) + 0{,}3478 \cdot g(0.8611)\right]$$

$$= 3\left[0{,}3478 \cdot f(1{,}4166) + 0.6521 \cdot f(2{,}9801) + 0.6521 \cdot f(5.0199) + 0.3478 \cdot f(6{,}5834)\right]$$

$$= 3\left[11{,}95 + 408{,}4 + 157165{,}5 + 559339\right]$$

$$= 720600$$

3.— Tenemos $h = 1$; $n = 3 \Rightarrow$ 0 1 2 3

TRAPECIO

$$\frac{h}{2}\left[f(x_0) + 2\left(f(x_1) + f(x_2)\right) + f(x_3)\right] =$$

0.

$$\frac{1}{2}\cdot\left[0.5 + 2(1 + 1.2) + 1\right] = \Rightarrow \boxed{V = \frac{2.95}{3} \approx 0.9833}$$

$$\frac{1}{2}\left[0.5 + 4.4 + 1\right] = \frac{5.9}{2} = 2.95$$

SIMPSON 3/8

$$\frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] = \frac{3}{8}\cdot\left[0.5 + 3(1 + 1.2) + 1\right] =$$

$$= \frac{3}{8}\left[0.5 + 6.6 + 1\right] = \frac{3\cdot 8.1}{8} = 3{,}0375$$

$$\Rightarrow \boxed{V = \frac{3{,}0375}{3} = 1{,}0125}$$

b) $p(x) = \frac{1}{2}\rho V^2(x)$ $\quad \rho = 1.29 \Rightarrow p(x) = 0{,}645\cdot V^2(x)$

$$\bar{P} = \frac{1}{3}\int_0^3 \frac{1}{2}\rho v^2(x)\,dx = 0.215\int_0^3 v^2(x)\,dx$$

v

x	0	1	2	3
$v^2(x)$	0.25	1	1.44	1

TRAPECIO: $\bar{P} = 0.215 \cdot \frac{h}{2}\left[0.25 + 2(1+1.44) + 1\right] = 0,6590$

Simpson 3/8 $\bar{J} = 0,215 \cdot \frac{3h}{8}\left[0.25 + 3\cdot 1 + 3\cdot 1,44 + 1\right] = 0,6910$

c) Gauss: No conocemos la expresión para $v(x)$, por tanto, sólo podemos evaluar $v(x)$ en los puntos que queramos. Sólo podríamos calcular el polinomio interpolador que pase por esos nodos y usar el polinomio en los nodos

$$P_v(x) = \frac{1}{2} + \frac{37}{60}x - \frac{1}{10}x^2 - \frac{1}{60}x^3$$

$$\int_0^3 v(x)\,dx = \int_0^3 P_v(x)\,dx = \frac{3}{2}\int_{-1}^{1} P_v\left(\frac{3}{2}\xi + \frac{3}{2}\right)d\xi =$$

$$\frac{3}{2}\cdot\left[\frac{5}{9}P_v\left(-0,7746\cdot\frac{3}{2}+\frac{3}{2}\right) + \frac{8}{9}P_v(3/2) + \frac{5}{9}P_v\left(0,7446\cdot\frac{3}{2}+\frac{3}{2}\right)\right] =$$

$$\frac{3}{2}\cdot\left[\frac{5}{9}P_v(0,3381) + \frac{8}{9}P_v(1.5) + \frac{5}{9}P_v(2,6619)\right] =$$

$$\frac{3}{2}\cdot\left[\frac{5}{9}\cdot 0,6964 + \frac{8}{9}\cdot 1,1438 + \frac{5}{9}\cdot 1,1186\right] =$$

$$\frac{3}{2}\cdot\left[0,3869 + 1,0167 + 0,6214\right] = \frac{3}{2}\cdot(2,0250) = 3,0375$$

$$\boxed{\bar{V} = \frac{1}{3}\cdot 3,0375 = 1,0125}$$

Coincide con Simpson 1/3, puesto que ambos métodos son exactos para polinomios de grado 3

4.– $R = [1,3] \times [2,5]$ $\quad f(x,y) = x^3 \operatorname{sen}(y)$

$$\int_a^b \int_c^d f(x,y)\,dx \cdot dy \;;$$

Usando Teorema Fubini

$$\int_a^b \int_c^d f(x,y)\,dx\,dy = \int_a^b dx \cdot \left[\int_c^d f(x,y)\,dy \right]$$

Calculamos la 1ª integral, para x fijas y haciendo cambio

$$y = \frac{d-c}{2} \cdot \alpha + \frac{d+c}{2} \Rightarrow dy = \frac{d-c}{2} \cdot d\alpha$$

$$\int_c^d f(x,y)dy = \left(\frac{d-c}{2}\right) \int_{-1}^{1} \underbrace{f\left(x, \frac{d-c}{2}\alpha + \frac{d+c}{2}\right)}_{g(x,\alpha)} d\alpha =$$

$$\left(\frac{d-c}{2}\right) \cdot \int_{-1}^{1} g(x,\alpha)\,d\alpha =$$

$$= \left(\frac{d-c}{2}\right) \cdot \left(g\left(x, \frac{-1}{\sqrt{3}}\right) + g\left(x, \frac{1}{\sqrt{3}}\right) \right) = A(x)$$

Luego calculamos la 2ª integral, haciendo el cambio

$$x = \frac{b-a}{2}\beta + \frac{b+a}{2} \Rightarrow dx = \left(\frac{b-a}{2}\right) d\beta$$

$$\int_{1}^{b} dx \cdot \left[\int_{c}^{d} f(x,y)\,dy\right] = \int_{a}^{b} A(x)\,dx =$$

$$\left(\frac{b-a}{2}\right)\cdot\int_{-1}^{1} A\Big(\underbrace{\left(\frac{b-a}{2}\right)\beta + \left(\frac{b+a}{2}\right)}_{h(\beta)}\Big)\, d\beta =$$

$$= \left(\frac{b-a}{2}\right)\left(h\left(-\frac{1}{\sqrt{3}}\right) + h\left(\frac{1}{\sqrt{3}}\right)\right)$$

Teniendo en cuenta el valor de h(x)

$$\left(\frac{b-a}{2}\right)\cdot\Bigg[A\left(\left(\frac{b-a}{2}\right)\cdot\left(-\frac{1}{\sqrt{3}}\right) + \left(\frac{a+b}{2}\right)\right)$$

$$+ A\left(\left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{a+b}{2}\right)\right)\Bigg]$$

y usando $A(x)$

$$\left(\frac{b-a}{2}\right)\cdot\left[\left(\frac{d-c}{2}\right)\left(g\left(\left(\frac{b-a}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\frac{-1}{\sqrt{3}}\right)\right.\right.$$

$$\left.+g\left(\left(\frac{b-a}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\frac{1}{\sqrt{3}}\right)\right)$$

$$+\left(\frac{d-c}{2}\right)\cdot\left(g\left(\left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\frac{-1}{\sqrt{3}}\right)\right.$$

$$\left.\left.+g\left(\left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\frac{1}{\sqrt{3}}\right)\right)\right]$$

y usando $g(x)$, sacando $\left(\frac{d-c}{2}\right)$ factor común

$$\left(\frac{b-a}{2}\right)\left(\frac{d-c}{2}\right)\cdot\left[f\left(\left(\frac{b-a}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\left(\frac{d-c}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)+\left(\frac{d+c}{2}\right)\right)+\right.$$

$$+f\left(\left(\frac{b-a}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\left(\frac{d-c}{2}\right)\left(\frac{1}{\sqrt{3}}\right)+\left(\frac{d+c}{2}\right)\right)+$$

$$+f\left(\left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\left(\frac{d-c}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)+\left(\frac{d+c}{2}\right)\right)+$$

$$\left.+f\left(\left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right)+\left(\frac{a+b}{2}\right),\left(\frac{d-c}{2}\right)\left(\frac{1}{\sqrt{3}}\right)+\left(\frac{d+c}{2}\right)\right)\right]$$

Para $a=1$ $b=3$ $\Rightarrow \frac{b-a}{2} = \frac{3-1}{2} = 1$;

$\frac{b+a}{2} = \frac{3+1}{2} = 2$

Para $c=2$ $d=5$ $\Rightarrow \frac{d-c}{2} = \frac{5-2}{2} = 3/2$;

$\frac{d+c}{2} = \frac{5+2}{2} = 7/2$

La fórmula queda

$$\frac{3}{2}\cdot\left[f\left(-\frac{1}{\sqrt{3}}+2;\ -\frac{3}{2\sqrt{3}}+7/2\right)+f\left(-\frac{1}{\sqrt{3}}+2,\ \frac{3}{2\sqrt{3}}+7/2\right)\right.$$

$$\left.+f\left(\frac{1}{\sqrt{3}}+2,\ \frac{-3}{2\sqrt{3}}+7/2\right)+f\left(\frac{1}{\sqrt{3}}+2,\ \frac{3}{2\sqrt{3}}+\frac{7}{2}\right)\right]$$

con $f(x,y) = x^3\cdot \operatorname{sen}(y)$

$$\frac{3}{2}\cdot\left[3996 + -2(7084) + 8.3223 + (-16.1039)\right]$$

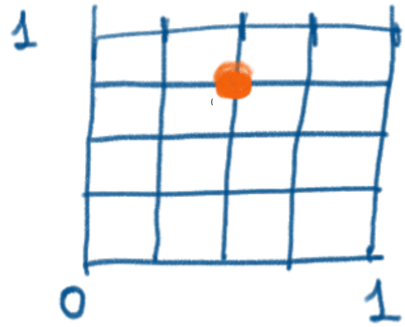
$$= \frac{3}{2}\cdot(-9.0903) \approx -13{,}635.$$

EXACTA: $2\cdot(\cos(2)-\cos(5)) = -13{,}996180$

7.- $f(x,y) = x^3 \cdot sen(y)$ $\quad R = [0,1] \times [0,1]$

Discretización $\quad x = [0, 1/4, 2/4, 3/4, 1]$

$y = [0, 1/4, 2/4, 3/4, 1]$



$$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{f(x_0+h, y_0) - f(x_0-h, y_0)}{2h}$$

$x_0 = 0.5$; $y_0 = 0.75$; $h = 0.25$

a) $$\frac{\partial f}{\partial x}(0.5, 0.75) = \frac{f(0.75, 0.75) - f(0.25, 0.75)}{2(0,25)}$$

$$= \frac{0.2876 - 0.0107}{0.5} = 0.5538$$

$\frac{\partial f}{\partial x} = 3x^2 \cdot sen(y)$ $\quad \frac{\partial f}{\partial x}(2/4, 3/4) = 0.5112$

b) $$\frac{\partial f}{\partial y}(0.5, 0.75) = \frac{f(0,5; 1) - f(0.5, 0.5)}{2 \cdot (0.25)} = 0.090511$$

$\frac{\partial f}{\partial y} = x^3 \cos(y) \Rightarrow \frac{\partial f}{\partial y}(1/2, 3/4) = 0.091461$

c) $\frac{\partial^2 f}{\partial x^2}(0.5, 0.75) = \frac{f(x_0+h, y_0) - 2f(x_0, y_0) + f(x_0-h, y_0)}{h^2} =$

$= \frac{f(0.75, 0.75) - 2f(0.5, 0.75) + f(0.25, 0.75)}{(0.25)^2} =$

$= 2.0449$

EXACTA $\Rightarrow$ $\frac{\partial^2 f}{\partial x^2}(x, y) = 6x \cdot sen(y) \Rightarrow$

$\frac{\partial^2 f}{\partial x^2}(0.5, 0.75) = 2.0449$

d)

$\frac{\partial^2 f}{\partial^2 y}(0.5, 0.75) \simeq \frac{f(x_0, y_0+h) - 2f(x_0, y_0) + f(x_0, y_0-h)}{h^2}$

$= \frac{f(0.5, 1) - 2f(0.5, 0.75) + f(0.5, 0.5)}{(0.25)^2} =$

$= -8.4762 \times 10^{-2}$

EXACTA:

$$\frac{\partial^2 f}{\partial y^2}(x,y) = -x^3 \operatorname{sen}(y) \Rightarrow$$

$$\frac{\partial^2 f}{\partial y^2}(0.5, 0.75) = -8.520484 \times 10^{-2}$$

e) $\nabla f(0.5, 0.75)$

$$= \left[\frac{\partial f}{\partial x}(0.5, 0.75), \frac{\partial f}{\partial y}(0.5, 0.75)\right]$$

$$= [0.5538, 0.090511]$$

$$\Delta f(0.5, 0.75) =$$

$$\frac{\partial^2 f}{\partial x^2}(0.5, 0.75) + \frac{\partial^2 f}{\partial y^2}(0.5, 0.75) =$$

$$= [2.0449 + (-8.4762 \times 10^{-2})]$$

$$= 1.960154$$

f) $\frac{\partial^2 f}{\partial x \partial y}(0.5, 0.75) = \frac{\partial}{\partial y}\left[\frac{\partial f}{\partial x}\right](0.5, 0.75) =$

$$= \frac{\partial f/\partial x\,(0.5, 1) - \partial f/\partial x\,(0.5, 0.5)}{2 \cdot 0.25} =$$

$$= \frac{\frac{f(0.75,1) - f(0.25,1)}{2(0.25)} - \frac{f(0.75,0.5) - f(0.25,0.5)}{2\cdot(0.25)}}{2\cdot(0{,}25)}$$

$$= \frac{f(0.75,1) - f(0.25,1) - f(0.75,0.5) + f(0.25,0.5)}{4\cdot(0.25)^2}$$

$= 0.58823$

EXACTA $\quad \frac{\partial^2 f}{\partial x \partial y}(x, y) = 3x^2 \cos(y)$

$\Rightarrow \frac{\partial^2 f}{\partial x \partial y}(0.5, 0.75) = 0.54876$