A family of third order Steffensen's methods on Banach spaces

Determining the zeros of a function has attracted the attention of pure and applied mathematicians for centuries. Solving equations numerically is a classical problem. General problems may be formulated in terms of finding zeros. The roots of a nonlinear equation cannot in general be expressed in closed form. Thus, in order to solve nonlinear equations, we have to use approximate methods. One of the most important techniques to study these equations is the use of iterative processes, starting from an initial approximation x_0, called pivot, successive approaches (until some predetermined convergence criterion is satisfied) x_i are computed , $i = 1, 2, \dots$, with the help of certain iteration function $\Phi : E \to E$:

$$x_{i+1} := \Phi(x_i), \quad i = 0, 1, 2 \dots \tag{1}$$

The advance of computational techniques has allowed the development of some more complicated iterative methods in order to obtain greater order of convergence.

For locating the root of a operator Steffensen's method is an iterative process achieving quadratic convergence without employing derivatives.

In this lecture, we present a family of third order generalized Steffensen's type methods. The main adventage of these methods is they do not need evaluate any derivative, but having the same properties of convergence than the classical third order methods. The methods will depend, in each iteration, of a parameter α_n. These parameters will be a control of the good approximation to the derivatives. We will study their convergence and we will test their competitivity with respect the classical methods. They seemed to work very well in our preliminary numerical results.