Hörmander and Lojasiewicz proved that for each polynomial $P$ and each (tempered) distribution $T$ there exists a (tempered) distribution $S$ such that $T = PS$. The division problem in the space $S'(^\infty \mathbb{R}^n)$ of tempered distributions on $\mathbb{R}^n$ can be stated as follows: Let $F$ be a multiplier of the space $S(\mathbb{R}^n)$ of rapidly decreasing functions, i.e. a smooth function $F$ satisfying $FS(\mathbb{R}^n) \subset S(\mathbb{R}^n)$. Find conditions on $F$ to ensure that for each tempered distribution $T \in S'(\mathbb{R}^n)$ there is a tempered distribution $S$ such that $T = FS$. It is known that $F \in \mathcal{E}(\mathbb{R}^n)$ is a multiplier in $S(\mathbb{R}^n)$ if and only if for each $k \in \mathbb{N}$ there exist $C > 0$ and $j \in \mathbb{N}$ such that $|F^{(\alpha)}(x)| \leq C(1 + |x|^2)^j$ for each multiindex $\alpha$ with $|\alpha| \leq k$. Here $|x|$ denotes the Euclidean norm on $\mathbb{R}^n$. The space of multipliers on $S(\mathbb{R}^n)$ is denoted by $\mathcal{O}(\mathbb{R}^n)$ and $\mathcal{O}_M$ in case $n = 1$.

A multiplier $F \in \mathcal{O}_M(\mathbb{R}^n), F \neq 0$, gives a positive solution for the division problem in $S'(\mathbb{R}^n)$ if and only if the multiplication operator $M_F : S(\mathbb{R}^n) \to S(\mathbb{R}^n), f \to Ff$, has closed range $\text{rg}(M_F)$. If $F \in \mathcal{E}(\mathbb{R}^n)$ is an arbitrary smooth function, the division problem for distributions is also equivalent to the fact that the multiplication operator $M_F : \mathcal{E}(\mathbb{R}^n) \to \mathcal{E}(\mathbb{R}^n)$ has closed range. The characterization for arbitrary dimension seems to be still open. However, in the one variable case, it was already known by Schwartz that a smooth function $F \in \mathcal{E}(\mathbb{R})$ satisfies that $M_F : \mathcal{E}(\mathbb{R}) \to \mathcal{E}(\mathbb{R})$ has closed range if and only if all $F$ has only isolated zeros of bounded order. Although there is a close relation between the two cases and they are equivalent in case $F$ is a polynomial, there is no analog characterization of those multipliers $F \in \mathcal{O}_M$ such that the range of $M_F : S(\mathbb{R}) \to S(\mathbb{R})$ is closed. This is the question we consider in this work. We get a characterization of the the multipliers $F \in \mathcal{O}_M(\mathbb{R}), F \neq 0$, such that the operator $M_F : S(\mathbb{R}) \to S(\mathbb{R})$ has closed range in terms of the zeros of $F$ in the one variable case.

This is a joint work with J. Bonet and L. Frerick.