Curso 2021/2022
Grado en Ingeniería Química Industrial
Matemáticas II - Problemas Temas 1
Cálculo integral en una variable
Cálculo de primitivas

1. Calcula las siguientes integrales indefinidas inmediatas

(a) $\int (2x-1)^3\, dx$ (b) $\int (4x+3)\, e^{-\left(2x^2+3x+1\right)}\, dx$ (c) $\int \frac{\log x}{x}\, dx$

(d) $\int \frac{x}{2x^2+3}\, dx$ (e) $\int \left(x+x^2\right)^{-1/2}(1+2x)\, dx$ (f) $\int \operatorname{sen}(ax)\cos(ax)\, dx$

2. Calcula las siguientes integrales por el método de cambio de variable, usando el cambio adecuado en cada caso:

(a) $\int \frac{e^x-1}{e^x+2} dx$ (b) $\int \frac{5^x+2}{5^x-1}\, dx$ (c) $\int \frac{1+\operatorname{arcsen}^2 x}{(1+\operatorname{arcsen} x)\sqrt{1-x^2}}\, dx$

(d) $\int \frac{1}{e^{2x}-e^x}\, dx$ (e) $\int \frac{\tan^3 x}{1+\tan^2 x}\, dx$ (f) $\int \frac{\arctan x+3}{(2-\arctan x)\left(1+x^2\right)}\, dx$

3. Calcula las siguientes integrales por el método de integración por partes

(a) $\int x\cos(3x)\, dx$ (b) $\int x^2\operatorname{sen}(5x)\, dx$ (c) $\int e^{-x}\operatorname{sen}(2x)\, dx$

(d) $\int \log x\, dx$ (e) $\int x^2e^{-x}\, dx$

4. Calcula las siguientes integrales trigonométricas donde $a, b \in \mathbb{R}$:

(a) $\int \frac{1}{\operatorname{sen}^2 x\cdot\cos x}\, dx$ (b) $\int \operatorname{sen}(ax)\cos(bx)\, dx$ (c) $\int \operatorname{sen}(ax)\operatorname{sen}(bx)\, dx$

(d) $\int \cos(ax)\cos(bx)\, dx$ (e) $\int \cos^2(ax)\, dx$ (f) $\int \operatorname{sen}^2(ax)\, dx$

(g) $\int \cos^5(x)\, dx$ (h) $\int \operatorname{sen}^4(x)\, dx$ (i) $\int \frac{1}{5-3\cos x+4\operatorname{sen} x} dx$

(j) $\int \frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}\, dx$ (k) $\int \frac{1}{10+6\operatorname{sen} x+8\cos x} dx$ (l) $\int \tan^3 x dx$

(m) $\int \frac{\tan x}{1+\operatorname{sen} x} dx$ (n) $\int \operatorname{sen}^4 x\cos^2 x dx$ (o) $\int \operatorname{sen}^2 x\cos^2 x dx$

5. Calcula las siguientes integrales de funciones racionales:

(a) $\displaystyle\int \frac{1}{(x-2)(3-x)} dx$ (b) $\displaystyle\int \frac{5x+1}{x^3-3x+2}\, dx$ (c) $\displaystyle\int \frac{1}{x^2+2x+5} dx$

(d) $\displaystyle\int \frac{1}{4x^2+4x+3}\, dx$ (e) $\displaystyle\int \frac{x+1}{x^2+x-2}\, dx$ (f) $\displaystyle\int \frac{1}{x^2+10x+16}\, dx$

(g) $\displaystyle\int \frac{x+2}{(x-1)(x+3)}\, dx$ (h) $\displaystyle\int \frac{3x^2+2x+7}{x^2+1}\, dx$ (i) $\displaystyle\int \frac{2x^3+9x^2+4}{x^2(x^2+4)(x-1)}\, dx$

(j) $\displaystyle\int \frac{x}{x^2+4x+5}\, dx$ (k) $\displaystyle\int \frac{x^5-3x^4-2x^3-7x^2+x-4}{x^3+3x^2-x-3}\, dx$ (l) $\displaystyle\int \frac{1}{4x^2-4x+4}\, dx$

(m) $\displaystyle\int \frac{1}{x^2+2x+4}\, dx$ (n) $\displaystyle\int \frac{x-3}{x^2+2x+2}\, dx$ (o) $\displaystyle\int \frac{x+1}{x^2-3x+5}\, dx$

(p) $\displaystyle\int \frac{33x^2-3x-6}{(9x^2-1)(2x+1)}\, dx$ (q) $\displaystyle\int \frac{1}{x^2+4x+2} dx$ (r) $\displaystyle\int \frac{1}{(x^2+1)^3} dx$

(s) $\displaystyle\int \frac{3x^3+2x^2+x-7}{x^4+x^3-3x^2-5x-2}\, dx$ (t) $\displaystyle\int \frac{4}{(x-1)^2(x^2+1)^2} dx$ (u) $\displaystyle\int \frac{1}{\left((2x+1)^2+1\right)^2} dx$

(v) $\displaystyle\int \frac{9}{(x^2+x+1)^2(x-1)^2} dx$ (w) $\displaystyle\int \frac{x^4+x^3-12x^2-25x-5}{x^3-7x-6}\, dx$ (x) $\displaystyle\int \frac{1}{(x-1)(x^2+x+1)^2} dx$

6. Calcula las siguientes integrales de funciones irracionales algebráicas:

(a) $\displaystyle\int \frac{\sqrt{x^3}}{x-1}\, dx$ (b) $\displaystyle\int \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}-1}\, dx$ (c) $\displaystyle\int \frac{\sqrt{x}+1}{\sqrt{x^3}-1}\, dx$

(d) $\displaystyle\int \frac{\sqrt[5]{x}}{\sqrt[5]{x}+1}\, dx$ (e) $\displaystyle\int \frac{x+\sqrt{(x+1)^3}}{(x-1)\sqrt{x+1}} dx$ (f) $\displaystyle\int \frac{\sqrt[3]{x-1}-1}{\sqrt[3]{(x-1)^2}+1} dx$

(g) $\displaystyle\int x\sqrt{\frac{x+2}{x-2}} dx$ (h) $\displaystyle\int \frac{\sqrt{(x-2)^3}-\sqrt{(x-2)^5}}{\sqrt{(x-2)}+3} dx$ (i) $\displaystyle\int \frac{1}{x}\sqrt{\frac{x-1}{x+1}} dx$

7. Calcula las siguientes integrales de funciones radicales:

(a) $\displaystyle\int \frac{1}{\sqrt{-x^2-2x+3}}\,dx$ (b) $\displaystyle\int \frac{1}{\sqrt{-x^2+2x+1}}\,dx$ (c) $\displaystyle\int \frac{1}{\sqrt{-4x^2+4x+1}}\,dx$

(d) $\displaystyle\int \frac{x}{\sqrt{-x^2-4x+2}}\,dx$ (e) $\displaystyle\int \sqrt{-4x^2-8x-3}\,dx$ (f) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$

(g) $\displaystyle\int \sqrt{4x^2+8x+8}\,dx$ (h) $\displaystyle\int \sqrt{-x^2+4x}\,dx$ (i) $\displaystyle\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

(j) $\displaystyle\int \sqrt{x^2-2x-3}\,dx$ (k) $\displaystyle\int \sqrt{-x^2+2x}\,dx$ (l) $\displaystyle\int \frac{x^2}{\sqrt{3x-x^2}}\,dx$

(m) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+x-3}}\,dx$ (n) $\displaystyle\int \frac{1}{\left(x-\frac{1}{2}\right)\sqrt{4x^2-1}}\,dx$ (o) $\displaystyle\int \frac{1}{\sqrt{4x^2+x+1}}\,dx$

(p) $\displaystyle\int \frac{1}{x\sqrt{-x^2+2x+1}}\,dx$ (q) $\displaystyle\int \frac{1}{x\sqrt{-2x^2-x+1}}\,dx$ (r) $\displaystyle\int \frac{1}{(x-1)\sqrt{-x^2+x+4}}\,dx$

(s) $\displaystyle\int \frac{1}{\sqrt{x^2+3x+2}}\,dx$ (t) $\displaystyle\int \frac{1}{x\sqrt{x^2-4}}\,dx$ (u) $\displaystyle\int \frac{1}{\sqrt{x^2+x-2}}\,dx$

8. Aplica el método Alemán para calcular las siguientes integrales:

(a) $\displaystyle\int \frac{3x+1}{\sqrt{-x^2-2x+1}}\,dx$ (b) $\displaystyle\int \frac{-12x^3+2x^2+8x+7}{\sqrt{-4x^2+4x+4}}\,dx$ (c) $\displaystyle\int \frac{-2x^2-5x}{\sqrt{-x^2-2x+1}}\,dx$

(d) $\displaystyle\int \frac{2x^2+2x-6}{\sqrt{x^2+2x-3}}\,dx$ (e) $\displaystyle\int \frac{4x^2+3x-1}{\sqrt{x^2+x}}\,dx$ (f) $\displaystyle\int \frac{4x^2-x-7}{\sqrt{x^2-x-4}}\,dx$

9. Calcula las siguientes integrales binomiales

(a) $\displaystyle\int \frac{1}{x^3\sqrt[3]{x^3+1}}\,dx$ (b) $\displaystyle\int \frac{x^2}{\sqrt[3]{(2+x)^2}}\,dx$ (c) $\displaystyle\int \sqrt{\frac{1+2x^{-1}}{x}}\,dx$

(d) $\displaystyle\int \frac{\sqrt[3]{x^2}}{\sqrt[6]{2+x^{-2}}}\,dx$ (e) $\displaystyle\int \frac{\sqrt{x}}{2+\sqrt{x}}\,dx$ (f) $\displaystyle\int \frac{1}{x^{\frac{2}{3}}\sqrt{1+x^{-\frac{1}{3}}}}\,dx$